PHOTOGRAPHIC PHOTOMETRY

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PHOTOGRAPHIC PHOTOMETRY

A STUDY OF
METHODS OF MEASURING RADIATION
BY PHOTOGRAPHIC MEANS

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G. M. B. D.
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PREFACE

It is remarkable that the intensities of the simple stimuli, which are immediately perceived by the senses, are difficult to compare with any accuracy. While the pitch of a sound may be decided exactly, it is much harder to give a measure of its volume. In the same way, the frequency of light is very much easier to determine than its intensity.

Fundamentally there are only two ways of measuring the intensity of light. The one is to determine the rise in temperature caused by light on being absorbed by a black body. The other is to measure the number of electrons removed by light from stable orbits in the atoms. The first method is in principle the more general, as it gives a direct measure of the energy. The second method is perhaps more elegant, as it resolves itself into counting quanta. In practice the first method is so inconvenient that it has been superseded almost entirely by the second. For while the first method requires the elimination of the most minute fluctuations of temperature, the second

only requires the elimination of extraneous light.

There are two ways of counting quanta; one can determine electrically the number of electrons emitted under the influence of the light, or one can determine chemically or otherwise the number of atoms ionized by the light. The photographic plate or film is nowadays so valuable an article of commerce that its production and use have been standardized, and it provides far and away the most convenient material for this latter purpose. The silver-halide in the gelatine is distributed in grains on the plate. If one atom in a grain has absorbed a quantum and been ionized, the whole grain is rendered developable. In principle therefore all that one need do is to count the developed grains.

In practice the method is far more complicated. Though the darkening of the plate has been used for years as a measure of the intensity of the incident light, the finer details of photographic photometry appear never to have been examined. It is thus fortunate that the authors, who have spent much time and research in the last few years in working out the best technique, should have put together their results in a monograph, which deals fully

with both the theoretical and practical sides of the points involved. Not only should it prove useful to those interested in the question from the purely practical point of view; it should also stimulate and provoke thought in all physicists who are examining the fundamental problems of the interaction of radiation and matter.

F. A. LINDEMANN.

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I

INTRODUCTION

1. *Object of Book.*

PHOTOGRAPHIC methods of measuring the intensity of light have recently been coming more and more into use for a number of purposes: in many cases such methods are the only ones possible to use, while in many others they are by far the most convenient. Thus, when measuring the relative brightness of spectral lines from a source which cannot be kept constant, all measurements must be made simultaneously, which can hardly be done except by obtaining their intensities from a photographic plate. Again, when working in the ultra-violet region of the spectrum, visual measurements cannot be made and photography is very convenient.

It seems, therefore, worth while to try to review the whole subject, discussing the principal methods employed, the sources of errors, how these errors can be minimized, and generally to find the best method of working. Various papers have dealt with individual points, but no general review seems to have been written. It is hoped that the

present notes may be of use to any one starting work in this subject, for the writers are painfully aware how much time would have been saved had they had, at the first, the information which has now been slowly acquired in the course of their researches.

As usually practised, photographic photometry cannot be relied upon to give greater accuracy than a 5 per cent. to 10 per cent. probable error. By taking proper precautions the probable errors should not be greater than about 1 per cent.

We shall discuss here the main principles of the two chief methods usually employed, and then go on to consider their various errors, and how they can be reduced; but it seems desirable first to deal with a few points which, though well known to workers in this subject, may not be familiar to any one using this method of photometry for the first time.

2. *Definition of the 'Density' of a Plate.*

The term 'density' is now used with a very definite meaning when applied to a photographic plate or filter, and is expressed by

$$D = \log_{10} \text{(incident light/transmitted light)}.$$

Thus a plate having a density of 0·0 transmits the whole light; while with a density of 1·0 only 10 per cent. is transmitted. Again, if two plates

differ in density by 1·0, the one will transmit ten times as much light as the other, whatever their absolute densities may be, while if their densities differ by, say, 0·0043, the one will transmit 1 per cent. more light than the other. To speak of a difference of density as a percentage of the absolute density has, therefore, little meaning.

It must be noted that the measured density of a plate will be different, according to whether we take the transmitted light to include only that light which is transmitted undeviated through the plate, or to include both this, and also all light scattered by the film, which comes out in all directions on the far side. For some purposes we require the one definition, and for some the other. Thus, when prints are made in contact with the negative, the whole of the transmitted light is effective, while if enlargements are made in a camera, little scattered light will be utilized. Hence the term 'contact density' has been applied where the whole light is used. Contact densities can be determined, if a completely scattering medium like an opal be placed against the plate, and the brightness of this measured. This, however, reduces the available light, and for most photometric purposes only directly transmitted light is used. Finally, if light be incident on the plate or filter at all angles to the normal, much of the light which is transmitted will have traversed a longer path in the absorbing material than the

normal thickness, and more light will be absorbed, thus leading to a higher effective density.

When there is appreciable background fog in a negative, the density of the image is usually taken as the difference between the measured densities of image and background, though this is not strictly accurate.*

3. *Methods of Reducing the Intensity of Light in a Known Ratio.*

In photometry it is frequently necessary to reduce the intensity of light in an accurately known ratio. Various methods are in use to which we may briefly refer.

Nicol Prisms. The beam of light is passed through two Nicol prisms, or other polarizing devices, one of which can be rotated. By adjusting the angle between the principal planes of the two prisms, any desired fraction may be transmitted, the amount transmitted being proportional to $\cos^2 \theta$, where θ is the angle between the principal planes of the Nicol prisms. Thus, for a small change in θ, the corresponding change in the intensity (I) is given by

$$\frac{dI}{I} = 2 \tan \theta \, d\theta,$$

and as $\tan \theta$ increases very rapidly for large values of θ, a small error in setting the angle

* See Ch. V, § 10.

produces a large error in the transmitted light when θ is large, i. e. when only a small fraction of the light is required to be transmitted.

The two parts of a Nicol prism are cemented together with Canada balsam, but as balsam is opaque to ultra-violet light such prisms cannot be used for work in this part of the spectrum. Foucault prisms, which have no cemented surfaces, however, are satisfactory, but have a very small field of view.

Variable Aperture. If the illumination is uniform the amount of light may be changed by varying the aperture of a lens system. To eliminate errors due to uneven illumination, a gauze or grating having opaque and clear spaces may be used. The ratio of clear to total area may be obtained by direct measurement.

Sector Wheel. These have frequently been used, but since they produce an intermittent illumination, and since changing the angle of the sector changes the relative length of the individual exposures, this method really varies the time of exposure rather than the intensity. Owing to the peculiar nature of the photographic plate, such a method is quite unsatisfactory, and may lead to entirely erroneous results.

Optical Wedge. Another device (which is very useful and convenient) is the optical wedge. This usually consists of a wedge-shaped piece of neutral grey glass cemented to a similar piece of

clear glass to form a parallel plate, or else of a very thin wedge of neutral grey gelatine between glass or quartz plates.

The theory of the optical wedge is as follows:

Consider first a layer of absorbing material of unit thickness, and let I_o be the intensity of radiation incident normally on the layer. Suppose that in passing through the layer I_o is reduced to AI_o, where A is some fraction depending on the material, and the wave-length of the

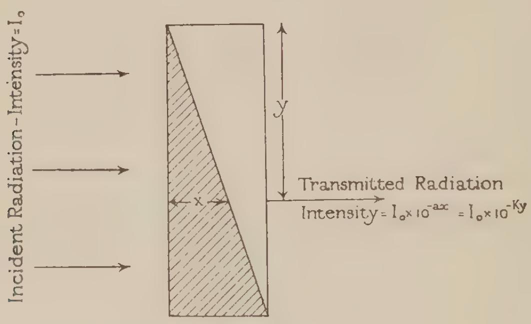

FIG. 1.

radiation. Each unit thickness of the medium reduces the intensity of the radiation which passes through it by the fraction A, so that after passing through x units I_o becomes $A^x I_o$; thus we may write

$$I = I_o \, 10^{-ax},$$

$$\text{where } a = \log_{10} \frac{1}{A}.$$

a is called the absorption coefficient of the medium for the given wave-length.

Now consider radiation passing normally through the wedge (Fig. 1).

At one extreme end of the wedge the thickness of absorbing material is zero (neglecting the absorption of the glass), so that the radiation passes through unchanged in intensity. But at a distance y from this end the radiation passes through a thickness x, which is proportional to y. Therefore

$$I = I_o \times 10^{-ax} = I_o \times 10^{-Ky},$$
$$\text{or } \log_{10} I = \log_{10} I_o - Ky. \quad . \quad . \quad . \ (1)$$

K is called the 'wedge constant'.

Optical wedges may be obtained which are nearly 'neutral', that is, the absorption coefficient of the glass or gelatine film of which they are made is almost the same for all visible wave-lengths. K is thus approximately constant for visible light, but increases for shorter wave-lengths, the increase generally beginning at about 4,000 Å.* The absorption by the compensating wedge of clear glass is very small for visible light. A reference line is generally ruled near the thin end of the wedge.

If, for any purpose, the transmission coefficient must be constant over a moderate area, and the change of density along the wedge would cause trouble, a second small fixed wedge may be placed

* A change of wedge constant with wave-length is of small importance, since it is easy to employ a different wedge constant for each wave-length.

behind the main wedge, with its density gradient in the opposite direction to that of the main wedge. If the wedge constant for the two wedges be the same, an area of uniform density will be produced, and its density may be changed by moving the main wedge, just as in the case when one wedge only is used.

For measurement in the ultra-violet region, glass wedges, of course, cannot be used. For wave-lengths greater than about 2,900 Å, gelatine wedges between quartz plates may be used. The absorption of these gelatine wedges increases rapidly with diminishing wave-length after about 4,000 Å,* so that the wedge constant at 3,000 Å is about 2·5 times that for visible light. Recently, Professor Merton † has made platinum wedges by cathode spluttering of platinum on quartz. These wedges are not generally uniform, but can be calibrated, so that this does not seriously matter. Such wedges can be used at least down to 2,200 Å.

4. *General Characteristics of a Photographic Plate.*

The general character of the relation between the light falling on a photographic plate and the density of the resulting image is shown by Fig. 2, where the density of the image is plotted as ordinate,

* Toy and Ghosh, *Phil. Mag.*, 1920.

† *Proc. Roy. Soc.*, 1924.

and the logarithm of the amount of light as abscissa. The actual form of the curves depends on many factors, such as the nature of the plate, the nature of the developer, the length of development, and the wave-length of the light. The middle part of the curve is often nearly straight, and the slope of

FIG. 2.

this part is important, and usually denoted by γ. With some modern plates, however, there are two straight portions of the curve, having different slopes. In this case the measurement of γ is somewhat indefinite, unless further specified.

Over the straight portion of the curve the density can be usefully expressed by

$$D = \gamma \log I t^{p} - i,$$

where I is the intensity of the light, t the time of exposure, and γ and i are approximately constant. The value of p (which is known as Schwarzschild's constant) is not strictly constant, but appears to change with the exposure. Messrs. Jones and Huse * have suggested the formula

$$D = \log (It) - a \sqrt{\left[\log \frac{I_o}{I}\right]^2 + 1},$$

where a and $\log I_o$ are constants. Their experiments also show that the slope (γ) of the characteristic curve for most plates is constant even though the time of exposure be increased by a factor of 10^5 (the intensity being reduced accordingly).

The value of i determines the 'inertia' of the plate. It should be noticed that this value is found by prolonging the straight portion of the curve until it cuts the 'log I' axis, and that it does not correspond to the minimum intensity which will give an appreciable image.

Increasing either the time of development or concentration of the developer increases the resulting density of the image, the relation roughly holding that

$$D_t = D_\infty (1 - e^{-kt})$$

or

$$\gamma_t = \gamma_\infty (1 - e^{-kt}),$$

where γ_t represents the value of γ for a development for time t, while γ_∞ represents the value for

* *Journ. Opt. Soc. Am.*, vol. ii, No. 4.

very long development, and similarly for the density D_∞ (the density due to chemical fog being allowed for—see Ch. V, § 10). The value k is known as the velocity-constant of development. The velocity of development decreases with age of the plate, so that old plates must be developed for a longer time.

The value of γ depends greatly on the wave-length of the light used, in general becoming smaller for shorter wave-lengths,* though Mr. Thorne Baker† finds evidence that the value of γ obtained with very long development is independent of wave-length.

The 'speed' of a plate is usually taken to be proportional to the reciprocal of the 'inertia'. The so-called 'H and D' (Hurter and Driffield) Number is equal to $\frac{34}{\text{inertia}}$, the inertia being found from plates developed under certain standard conditions and being expressed in terms of foot-candle-seconds.

Another curious characteristic of photographic plates is that, if a succession of very short isolated exposures are made on a plate, the resulting density is less than that which would have been

* Mr. G. R. Harrison (*Journ. Opt. Soc. Am.*, October 1925) finds that for many types of plate γ decreases slowly from 4,500 Å to 2,500 Å, often falling rapidly for shorter wave-lengths. The speed falls rapidly beyond 2,500 Å, but is nearly constant between 2,500 Å and 4,500 Å.

† *Trans. Faraday Soc.*, 1923.

obtained if the same total exposure had been given continuously. Thus any method of reducing the intensity which uses intermittent exposures (such as the sector wheel) should be avoided.

Fortunately for photographic photometry, the sensitivity of a plate does not appear to be affected greatly by the temperature of the plate at the time of exposure. Even at the temperature of liquid air the plate is still quite sensitive. Some experimenters have found evidence of an extension of sensitivity towards the red end of the spectrum when the plates are exposed while very hot.

It will at once be evident that the peculiar characteristics of a photographic plate, which are described above, make it necessary to be very careful what methods we employ in photographic photometry. Thus the fact that γ is different for different wave-lengths means that if two images formed by two beams of light of different wave-lengths produce equally dense images on a plate, then if we double the intensity of both beams of light the densities of the two images produced by them will no longer be equal (the photographic Purkinje effect). Hence, if it is necessary to compare the intensities of two beams of light of different wave-length, special precautions must be taken to guard against error.

5. *X-ray Photography.*

The effects of X-rays on the emulsion of a photographic plate are twofold. They not only act directly on the silver bromide, but in addition they excite the characteristic radiations of the silver and bromine atoms in the emulsion.* These effects are intensified by increasing the thickness of the gelatine film, and care must be taken to develop the film right through to the plate or other support. A slow developer should be used, as, with a very thick film, fogging of the surface of the film may take place before the developer has penetrated through the film.

The relation between the density of the photographic plate and the amount of radiation received by it may be represented by curves † resembling closely the 'characteristic curves' which, in the case of visible light, have been discussed in § 4. For X-rays the curved portion of the characteristic curve near the origin extends to higher densities than in the corresponding curves for light rays. Unfortunately in X-ray work, with correct exposure, i.e. on the straight-line part of the characteristic curve, the density is usually so great that printing from the plate or measurement of its density is

* M. de Broglie, *C. R.*, 1913–14.

† Tugman, *Journ. Ront. Soc.*, 11, 121, 1915. Hodgson, *B. J. P.*, 64, 654, 1917. Block and Renwick, *Trans. Far. Soc.*, 15, 2, 1920.

difficult. In general, X-ray photographs are taken partly or entirely on the under-exposure part of the curve, and a speed classification of X-ray plates, analogous to that devised by Hurter and Driffield for ordinary photographic plates, possesses no practical significance. In addition, the effect of different developers on the speed of plates is very marked.

As was pointed out in § 4, the blackening produced in time t by visible light of intensity I on a photographic plate may be regarded as proportional to log It^p, where p is approximately equal to 0·86. In the X-ray region the evaluation of p is difficult, but the work done so far appears to indicate that the value of p is very nearly unity.*

The experiments of Barkla and Martin † on the photographic effect of X-rays of varying wave-length show that, for long waves producing equal ionization in air, the photographic effect is practically constant until the radiation becomes of shorter wave-length than that characteristic of bromine X-ray radiation, when the effect increases. As the wave-length further diminishes, the effect approaches a constant, but higher value, until the wave-length approaches that corresponding to the characteristic X-ray radiation of silver. Here a

* Glocker and Franck, *Phys. Zeit.*, 22, 1921. Bowers, *Zeit. f. Phys.*, 14, 6, 374, 1923. Schuster, *Phys. Zeit.*, 24, 29, 1923.

† *Phil. Mag.*, 1913, vol. xxv, p. 296.

second marked increase takes place. The curve connecting the photographic effect and wave-length is very similar to that expressing the absorption of X-rays of different wave-lengths by silver bromide.

In conclusion it should be stated that the quantitative work carried out in X-ray photometry is neither as extensive nor as concordant in its results as that carried out in the photometry of visible light. Further, the need for using homogeneous radiation has not always been realized, so that the results obtained by different workers are of necessity difficult if not impossible of reconciliation.

II

STANDARD METHODS

1. *General.*

THE density of a photographic image depends not only on the exposure it has received, but also to a great extent on the time, temperature, &c., of development. In practice it is not feasible to make the development of all plates so nearly alike that the density of the image alone can be used as a measure of the intensity of the light falling on different plates. Further, even if development could be carried out with sufficient uniformity, one would be asking a great deal of the plate maker if one demanded that all plates of the same make should have identical properties. Moreover, it is well known that certain characteristics, such as the rate of development of a plate, change with the age of the plate.

For such reasons it is necessary to use the photographic plate to make a comparison of the intensities of two or more beams of light which give images on the same plate. The only strictly accurate method of procedure is to find positions

on two images on the same plate where the densities are identical. Then, provided the times of exposure were the same, and the wave-lengths of the lights employed identical, we may with confidence assume that the light incident on the plate was of the same intensity in the two cases, provided that the sensitivity and the development was uniform all over the plate.*

This is a fundamental principle in all trustworthy photographic photometry, and though in certain cases it is not possible to adhere to it strictly (as will be seen below), every modification introduces at the same time possible sources of error.

Photographic photometry has to be applied to all kinds of problems, but one of two general methods of working will be found applicable in nearly every case. Where we are measuring the intensities of various beams of light, it is convenient to allow the light to illuminate uniformly an optical wedge, an image of which is thrown on the plate. If two such images are formed by the two beams which have to be compared (the wave-lengths being the same), we shall have two images, each changing in density along its length. By means of a photometer we can then find places on the two images where the densities are equal, and can measure the difference in distance of these points from the reference line on the wedge. From this, knowing the wedge constant,

* See Ch. V, §§ 1 to 3.

we can at once find the ratio of the intensities of the two beams.

In certain cases it is necessary to make the results obtained from a number of plates all comparable with each other. In this case some source of light must be provided, which will remain accurately constant over the whole set of experiments, and an image obtained by means of this source must be formed on each plate, to be used as a standard for comparison.

It must be specially noted that no method of photographic photometry, without a special calibration of the plates,* can give the relative intensities of two beams of light of different wave-lengths, since the characteristics of the plates often differ considerably for different wave-lengths.† Where such a comparison is necessary it is best made by the aid of photographs of the spectrum of a source which radiates as a 'black' or 'grey' body of known temperature, so that the ratio of the energies in the various wave-lengths can be calculated.‡

* It is not always safe to assume that the calibration obtained with one batch of plates will apply accurately to another batch of the same make, nor even to other plates of the same batch if kept for any length of time.

† Ch. I, § 4.

‡ An arc between carbon electrodes, or even the hollow crater of the positive electrode, does not strictly radiate as a 'black' or as a 'grey' body. In its spectrum, lines and bands due to impurities and the surrounding atmosphere may be

In the description of Method A, given below, we have supposed that the results from all plates must be comparable. Where it is only necessary to compare the results from any one individual plate, the procedure can be somewhat simplified, and the 'standard' light image can be omitted.

The above method is probably the best and simplest in the majority of cases, but it cannot be employed for all problems. Thus in such a case as measuring the relative brightness of different parts of the corona, at a solar eclipse, a different method must be used. Here an ordinary photograph of the corona might be taken through suitable colour filters, to limit the range of wavelengths used. On the same plate an image would be formed of an optical wedge uniformly illuminated by a standard light, employing the same colour-filters, and keeping the time of exposure the same in all cases. Now by means of a photometer we measure the density of the image of the corona at any number of different places, and find the respective positions on the wedge image which have these densities. Thus again, knowing the wedge constant, we can calculate the relative

observed; the spectral intensities in the regions immediately adjacent to these lines obviously do not obey the 'black body' law of radiation. In general, however, isolated portions of the spectrum can be obtained which may be regarded as due to radiation from a black or a grey body, and these are sufficiently extended to enable accurate measurements to be made.

brightness of different parts of the corona. If the standard light illuminating the wedge is constant for all the plates, the results from different plates will be comparable, since plate factors and development factors have all been eliminated.

Which of these two methods should be employed must be settled by the nature of each particular problem. Both methods have been employed by the writers for different purposes, and both can be made to give very satisfactory and accurate results.

While the two methods outlined above are most suitable for general work, other methods have been used in special cases. Thus, for spectroscopic work, Professor Merton has used a coarse grating in the optical path, with the direction of the rulings parallel to the dispersion of the spectrograph. A very short slit is used, and each line is thus represented by a series of dots due to the spectra of different orders formed by the grating. The amount of light in the spectra of each order can be calculated, so that the number of dots corresponding to any one line which are visible is a measure of the intensity of that line. Naturally the method is not suitable where great accuracy is required, but may be convenient in certain cases. A similar method has been used for measuring star magnitudes. Here the grating is placed near the object-glass of the telescope, and each star is represented by a series of minute spectra.

In another method, chiefly suitable for measuring absorption coefficients, the main beam is divided into two, and a series of pairs of spectra are taken. One spectrum of each pair is taken with a beam which passes through a polarizer and analyser, the angle between the principal planes varying in each pair of the series. The second spectrum of each pair is taken with a beam which passes through the medium whose absorption coefficient is to be measured. The wave-lengths at which the various pairs of spectra have the same intensity are then estimated by eye so that an absorption curve can be plotted. This method seems to have no advantage over the wedge method.

The intensity of the incident light is sometimes made to vary logarithmically along a strip of the plate by means of special shutters and sector wheels instead of by an optical wedge. It is generally stated that this is done because the wedge constant cannot be trusted to be really constant. Good optical wedges have, however, very little variation, and in any case the wedge can be calibrated, both at different places and for different wave-lengths, with very little trouble.

2. *Method A* (1).

To make this description clearer we shall describe the method as applied to spectrographic

work. The necessary changes for other cases will be obvious.*

An optical wedge is placed either in front of the slit of the spectrograph or against the plate, with its lines of equal density parallel to the dispersion. The image, therefore, as it appears on the plate, decreases uniformly in density from top to bottom. It is, of course, necessary that the radiation falling on the wedge should be of uniform intensity over the whole height of the wedge. In practice the radiation does not usually fall on the wedge as a parallel pencil, so that it does not all pass through normally; it may be a convergent or divergent pencil. The effect of this is to give a different value of K; for this reason it is desirable to calibrate the wedge *in situ*, and not to take values of K measured in parallel light.†

Where it is required to make the results of one plate comparable with those of another plate, the sources of light whose intensities are to be measured are photographed through the wedge, and on the same plate a photograph is also taken with the same apparatus of a standard source, with an equal time of exposure.

After development the images are measured, and the distances from the zero line to the points where the density has a certain fixed value are

* Merton and Nicholson, *Phil. Trans. Roy. Soc.*, vol. 217 Slade and Toy, *Proc. Roy. Soc.*, xcvii, A, 1920.

† See Ch. II, § 5.

recorded*; so that for each image, and for any given wave-length, we know the value of y (equation (1) Ch. I, § 3) corresponding to a certain value of $\log I$. This equation shows that for a constant value of $\log I$, y is a measure of $\log I_o$, for

$$\log I_o = Ky + \log I.$$

Thus a number of different sources can be compared, wave-length for wave-length, with one another and with the standard source, by means of the appropriate values of y which have been found; for

$$\begin{aligned} \log I &= \log I_o - Ky \\ &= \log I_s - Ky_s, \end{aligned}$$

where I_s is the intensity of the radiation which falls on the wedge from the standard lamp, and y_s is the distance from the zero line to the point of given density at wave-length λ on the standard image.

$$\therefore \log (I_o/I_s) = K(y - y_s). \quad . \quad . \quad (2)$$

It will be observed that no characteristic of the plate or its treatment enters into this equation, and that I_o is measured in terms of a constant intensity, under the one condition that equal intensities give equal densities on the same plate.

The actual values of y measured are the distances from the reference line, but, as we are only concerned with differences, the fact that the reference line is not actually the line of zero density of the wedge does not matter.

* See Ch. IV, § 1, for discussion of best value to use.

3. *Method A* (2).

We have assumed that it is possible to obtain a standard source which will give sufficient density in the same time of exposure as that given to the other sources. This, however, is not always possible, as the writers found when making measurements of the sun's extreme ultra-violet light, and it may be necessary to give a longer exposure to the standard lamp. In this case the above theory has to be modified.

The law of blackening of the plate can be expressed approximately by the following relation * :

$$D = \gamma \log It^p - i.$$

Therefore for a constant density on any one plate, $\log It^p$ is constant, and

$$\log I_s + p \log t_s = \log I_s' + p \log t, \quad . \quad . \quad (3)$$

where t_s = time of exposure to standard lamp,
and t = time of exposure for experimental images.

I_s' is that intensity which would give the same density in t seconds as I_s actually gives in t_s seconds. In other words, I_s in equation (2) (Ch. II, § 2) has to be replaced by the equivalent intensity I_s', where

$$\log I_s' = \log I_s + p \log (t_s/t).$$

* See Ch. I, § 4. This equation is not true if the variables change over a large range. Over the small range of densities used in this class of work the errors will usually be unimportant.

The value of $p \log (t_s/t)$ is always the same for the same wave-length, if we assume that p is the same for all plates of the same make for a given wave-length. In the present state of our knowledge this seems a reasonable assumption.

4. *Method A* (3).

The writers found another modification of this method much more convenient on account of the great difficulty of obtaining a constant source of ultra-violet radiation.

The standard exposure is no longer made through the wedge in the spectrograph or camera. A small area of the plate is exposed to a uniform standard light, the intensity and time of exposure being adjusted to give a density about equal to that which is most suitable for measurement. The density at which the ordinates (y) are measured on each plate is then that of this standard patch. The measurements are made as before.

Assuming the law of the plate to be that previously given, we have

$$\begin{aligned} D &= \gamma (\log I_o - Ky + p \log t) - i \\ &= \gamma_s (\log I_s + p_s \log t_s) - i_s, \end{aligned}$$

where s refers to the standard lamp.

Thus at the standard density

$$\gamma (\log I_o - Ky + p \log t) = \gamma_s (\log I_s + p_s \log t_s) - (i_s - i).$$

$$\therefore \quad \log I_o = \left\{ \frac{\gamma_s}{\gamma} \log I_s - \frac{i_s - i}{\gamma} + \frac{\gamma_s p_s \log t_s - \gamma p \log t}{\gamma} \right\} + Ky$$

$$= A + Ky, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where A is a constant provided that the various quantities inside the bracket remain constant. In this way sources of light which have to be photographed on different plates can be compared, for

$$\log(I_o/I_o') = K(y-y'),$$

the unknown A disappearing.

This is a very useful method, but more assumptions have to be made than were necessary for the methods previously described. For instance, the value of $(i_s - i)$ may be different for two plates of the same make; and if two such plates are used, the values of A will be different and the plates will not be comparable, although there will be nothing to show that this is the case. The first method was designed to eliminate such variations as these.

Two important facts are indicated by equation (4), viz.

(1) There appears to be no reason for making t and t_s equal, since $(\gamma_s p_s \log t_s - \gamma p \log t)$ would not vanish.

(2) The wave-lengths given by the standard lamp need not be the same as any of those used for the other images. Nevertheless it is better

that they should be nearly equal, because in that case small differences which may occur in the constants of plates supposed to be identical will only introduce small errors. Thus, if the inertias of any two plates are slightly different, the difference in the values of $(i_s - i)$ for one plate and for the other will be smaller if $(i_s - i)$ is itself small, i. e. if λ_s and λ are nearly equal.

The physical meaning of the analysis which we have just made can be better understood by treating the subject graphically.

Let us assume that the characteristic curves of the plate are given,

(1) for the group of wave-lengths (λ_s) given by the standard lamp;

(2) for any wave-length (λ) whose intensity we are measuring. Let the straight part of these curves be represented by A and B respectively (Fig. 3), the ordinates being densities and the abscissae the values of $\log I$ (see equation (1), Ch. I, § 3). The value of I_s (the light for the standard lamp) is absolutely constant, and therefore the point J_s on the $\log I$ axis which corresponds to I_s is the same, no matter how the characteristic curves may vary. The density of the standard patch is now equal to d, and by measurement we find in what position on the experimental image (λ) the density is also d. If our assumptions (equation (4)) are correct, I, the value of the intensity of wave-length λ corresponding to the

standard density d, will always be the same, and $\log I_o$, which is equal to $(\log I + Ky)$, will be correctly measured; but if, for any reason, the value of I is not always the same, errors will be introduced and the values of $\log I_o$ obtained from different plates will not be strictly comparable. We shall now consider how I is likely to vary with changes in the constants of the plate.

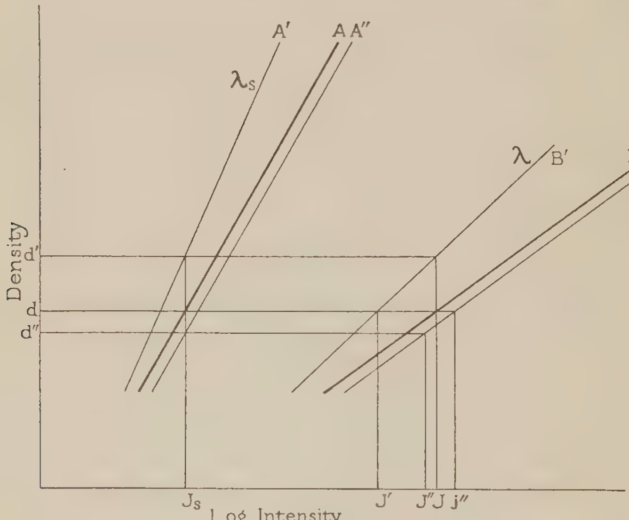

FIG. 3.

(a) If the development of the plate had been prolonged, or had been carried out at a higher temperature or with a stronger developer, we know that γ would have been larger, i. e. that the characteristic curves would have been steeper. Provided that points which were previously of

equal density are still of equal (though of course greater) density, the curves will then be as shown at A' and B', the standard density will be d', and the intensity of wave-length λ corresponding to d' will be I as before. Thus the effect of varying development is eliminated. If we had had no standard image, but had measured at a constant density d, the intensity corresponding to d would have been given by J'.

(b) Schwarzschild's constant, p, and the inertia may vary; supposing γ to remain constant, the curves will be moved to some such positions as A'' and B''. The exact extent of the movements will depend on the variations which occur, but if λ_s and λ are nearly equal it is probable that the two curves will be moved about equal distances.

In this event, d'' will be our standard density, and I'', represented by J'', the intensity corresponding to it. The intensity corresponding to d would be represented by j''. Whether the error given by JJ'' is less than that given by Jj'' depends upon the exact positions of the curves; though usually reduced, it is not necessarily eliminated.

We thus see that the most important function of the standard exposure is to eliminate the effect of varying development.

5. *Measurement of Wedge Constant in Method A.*

The measurement of the wedge constant is best made *in situ*, so that all conditions are exactly as they are in the actual experiment. All that is necessary is to take photographs with a constant source of light, and to interpose in alternate photographs a screen or other device, which cuts down the light in some known ratio. This screen is placed near one of the lenses, and may conveniently consist of a uniformly perforated gauze.

6. *Measurement of Images by Sharp Outline Method.*

In the above, we have supposed that the densities were measured by means of a photometer as described fully in a later section. Another method, however, has been used by Professor Merton, which, although it appears not to be so accurate, is simpler in that it does not involve the use of a photometer. A print of the original negative is made on a process plate, and is developed with a hard developer like alkaline hydroquinone, and the plate is slightly cut with a solution of potassium ferri-cyanide. From this positive another print is made by the same process, and so the series may be continued, each successive print becoming harder, and the edge of the wedge image sharper, until it can be measured

accurately under a travelling microscope, or by placing the negative in an enlarging lantern. By heavy development and cutting, a moderately hard outline may be obtained on the original negative if process plates are used. It will be noticed that all the errors due to uneven development and non-uniformity of the plates are magnified if many copies are made, but still the method seems capable of giving quite good accuracy.* When a travelling microscope is used, the plates are best illuminated during measurement by oblique, rather than direct light, as the edge is then somewhat sharper.

It is shown elsewhere that the best density at which to measure the photographic images is that represented by a point on the 'characteristic curve' of the plate, near the lowest part of its straight middle portion. A density of about 0·3 to 0·5 will generally fulfil these conditions, if the plate had been well developed. It follows, that if the original wedge image has a density of 0·5 at an ordinate of y cm. we should try to arrange the time of exposure when printing, so that on each successive print the density is also about 0·5 at an ordinate of y cm., and that finally on the last image the sharp outline which is measured is again at an ordinate of about y cm.

* Tests designed to indicate the relative accuracy of measurements made by a photometer and by the sharp outline method showed that the standard error was about twice as great with the latter method as with the former.

If this last method of measuring the images be used, and it is intended to adopt the method A (3), some changes must be made in the procedure, since we cannot now measure the ordinates where the density is exactly that of the standard patch. However, all that is necessary is to substitute a standard wedge image for the standard patch. Then, if we take differences between the ordinates of the experimental wedge images and those of standard wedge image, making allowance for any difference in wedge constants, the results of all plates will be comparable.

To avoid errors when making the prints, it is necessary that the illuminating light should be uniform. With an ordinary clear glass electric bulb, flaws in the glass, dust on the surface, and shadows of the filament and its supports cause surprisingly large irregularities in the light, even at a considerable distance. Further, it must be remembered that the eye does not easily detect such small changes if the changes are not abrupt.

7. *Standard Method B* (1).

In this method an ordinary photograph is taken, and the intensity of the light reaching each part of the image is determined from the density of the image. If it is necessary to make the results of two or more plates comparable, a photograph of some standard, unvarying source must be taken

on each plate for reference. The intensity of the light has sometimes been obtained directly from the density by assuming the law of the plate. It seems much better, however, to put on each plate the image of a standard optical wedge, illuminated by a standard source of light, the time of exposure being the same in all cases. The density of a certain part of the experimental image having been measured, the position on the wedge image is next found where the density is the same. The distance of this point from the reference line on the wedge image is measured. Since the densities, developments, and times of exposures are the same, it follows that these two parts must have received equal light, provided the wave-lengths are the same. If we know the wedge constant, we obtain at once the intensity of the light which fell on any part of the image, in terms of the standard source.*

It will be seen that there is no necessity for all densities to be within the limits between which the characteristic curve of the plate is straight.

This method of working is often very convenient, and gives good accuracy. Thus the authors employed it when measuring the brightness of different parts of the sun's disk for ultra-violet light with quite satisfactory results.

If very wide ranges of brightness have to be

* We believe that this method was first suggested by Professor Lindemann.

measured, the most intense lights would give inconveniently dense images. This can be obviated if a filter can be interposed which cuts down the light in an accurately known proportion.

8. *Method B* (2). *Modification when Lights of Different Wave-lengths are used.*

The times of exposure, and strictly the wave-length of the light used, should always be the same. Sometimes, however, as in the case of the ultra-violet work described in Ch. VII, § 2, no suitable source of the same wave-length is obtainable. In this case, provided the wave-lengths are not too different, little error will be introduced if we use an effective wedge constant, somewhat different from the true one. This effective wedge constant is found by taking two photographs of the experimental source of light, having reduced the intensity of the total light for one photograph in some known ratio. The distance on the wedge image between the two points corresponding to the densities of the two images allows the effective wedge constant to be calculated.

The theoretical effect of using a different wave-length for the experimental and standard images is easily seen as follows. In this, we assume the ordinary formula for the law of blackening, and also that Schwarzschild's constant (p) really is a constant, whereas it probably varies slightly with the density.

We have

$$D = \gamma \log It^{p} - i$$
$$= \gamma_s\{\log I_s - K_s y + p_s \log t_s\} - i_s,$$

where s refers to the standard lamp.

K_s = wedge constant for the wave-lengths given by the standard lamp,

y = distance along wedge.

Thus at a given value of D

$$\gamma(\log I + p \log t) = \gamma_s(\log I_s - K_s y + p_s \log t_s) - (i_s - i).$$

$$\therefore \log I = \left\{\frac{\gamma_s}{\gamma} \log I_s - \frac{i_s - i}{\gamma} + \frac{\gamma_s p_s \log t_s - \gamma p \log t}{\gamma}\right\} - \frac{\gamma_s K_s y}{\gamma}$$

$$= A - Ky, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where A = constant, K = the effective wedge constant = $K_s \gamma_s/\gamma$.

Thus we see that

(1) t and t_s need not be the same, and indeed there appears to be no advantage in making them equal, since we do not make $(\gamma_s p_s \log t_s - \gamma p \log t)$ vanish. We assume the various quantities inside the brackets are constant for all plates of the same make; otherwise a given value of y will represent different values of I on different plates.

(2) It does not greatly matter what wave-lengths are given by the standard lamp, except for the fact that in changing from one plate to another (of the same make) there is less likelihood of a change

in A if the wave-lengths of the standard and experimental lights are nearly equal.

The conclusions from this are the same as those which we found in the case of Standard Method A (3), § 4, and may be illustrated by an example. If we were measuring ultra-violet radiation, and our standard were an unscreened lamp whose maximum intensity was in the red or infra-red, a serious error (which we could not detect at once) might occur on account of a change in the red-sensitivity of the plate, unaccompanied by a corresponding change in the sensitivity to ultra-violet light. This would produce a change in $(i_s - i)$, which quantity might become appreciably smaller if the plate were for any reason more sensitive to red than those which we had hitherto been using; this would alter the constant A, with the result that, although we should not know it, our comparisons with other plates would be vitiated. If, however, we used only ultra-violet light for our standard (e. g. the band at about 3,600 Å transmitted by Chance's ultra-violet glass), any change in the sensitivity to these wave-lengths would, in all probability, be accompanied by an approximately equal change in the sensitivity to ultra-violet light of not very different wave-length.

We will now treat graphically the mathematical analysis which we gave above, first considering the determination of the effective wedge constant when using different wave-lengths.

Suppose, as we did in § 4, that we have the characteristic curves of a plate for the wave-lengths λ_s of the standard light and for the wave-length λ of the light which we are measuring.

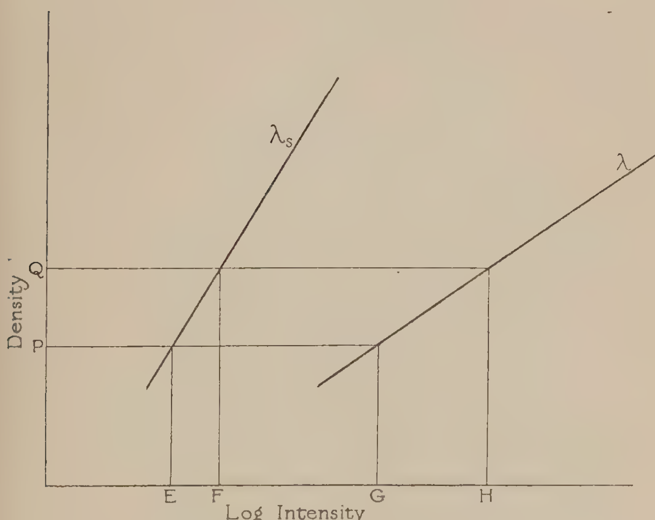

FIG. 4.

Take two points on the standard wedge image 1 cm. apart, and let P and Q be their densities (Fig. 4). Then the intensities which produced the densities P and Q were, by definition, in a ratio whose logarithm is K_s, i.e. $EF = K_s$. But the intensities of wave-length λ which would give the same two densities are not in the ratio K_s, but in another ratio whose logarithm is GH, and this

gives the effective wedge constant for wave-length λ, since $PQ = 1$ cm.

Thus $$K = K_s \times \frac{GH}{EF} = \frac{\gamma_s K_s}{\gamma},$$

as EF and GH vary as $1/\gamma_s$ and $1/\gamma$ respectively. In practice we do not deal with a single wave-length λ_s, but with a group whose limits and energy distribution are determined by the lamp, filter and plate used. Consequently γ_s is a mean value, and depends on the energy distribution; and it follows that the wedge constant K_s should be the same for all the wave-lengths used for the standard image; for if K_s varies over the range of wave-lengths of the light falling on the plate, the effective value of λ_s will change continuously over the length of the standard wedge image, and the effective wedge constant K will be different for different parts of this image. In this event the wedge would have to be calibrated at a number of points.

In the case referred to in § 7 the standard was a half-watt lamp and the wedge was of glass, and these combined to give a small amount of ultra-violet energy; so that the only region of the spectrum used was the visible and near ultra-violet, for which the wedge constant is uniform.

If it is desired to use ultra-violet light for the standard, a narrow band should be isolated to avoid variations in wedge constant.

Let us now consider the influence of changes in γ, i, and p, and as in the graphical consideration of

Standard Method A (3), § 4, in Fig. 5 let A and B be the characteristic curves for wave-lengths λ_s and λ respectively. An intensity I of wave-length λ, represented in the figure by J, falls on a given point on the plate and results in a density d, and it is found that this density occurs also at a distance

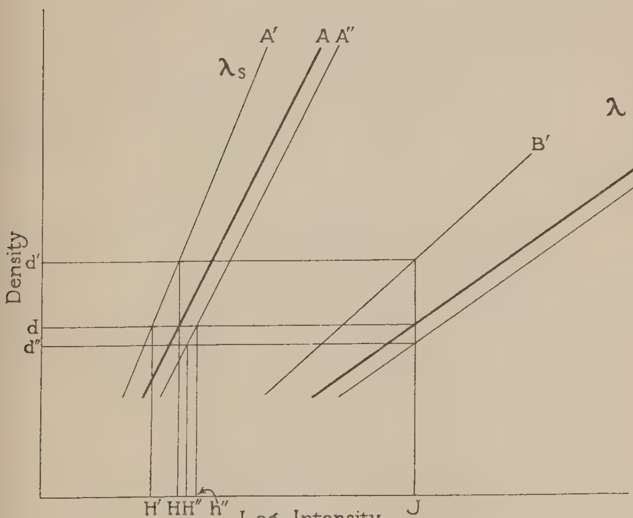

FIG. 5.

y from the zero line on the standard image. Therefore I was equivalent photographically to an intensity G_s of wave-length λ_s represented by H. The necessary and sufficient condition that our photometry shall be accurate is that G_s shall be constant for a given value of I, and shall not depend on the plate or its treatment; in other

words, that a given value of I shall always be expressed by the same value of y, since

$$\log G_s = \log I_s - Ky.$$

The extent to which this condition is likely to be fulfilled depends upon the same factors as those we discussed in § 4.

(*a*) Other things being equal, if the development be prolonged, the two points under consideration where the density was d will still be of equal density, but their density will be increased to d', and d' will still correspond to the intensity given by H. Measurements at a constant density d would have resulted in an error measured by HH'.

(*b*) A change in the inertia or in Schwarzschild's constant will move the curves parallel to themselves to positions such as A'' and B''. d'' is now the density resulting from intensity I, and the position of H has become H''. The error introduced by the measurement at density d would have been represented by Hh'' under the conditions represented in the figure, whereas if measured at the density d'' the error would be only that corresponding to HH''.

This last general method of working is often very convenient even for spectroscopic work. At times one is restricted to a very short slit, or one may want to measure variations in brightness along the slit. In such cases Method A of course cannot be used. There is no theoretical reason why Method B should not give as good accuracy

as Method A. It has the disadvantage that it requires a photometer for measurement, since the type of measurement described in § 6 cannot be used.

9. *Measurement of Star Magnitudes.*

It is impossible to deal fully here with the various photographic methods which have been used for measuring the brightness of stars.* Five chief methods have been adopted:

(i) By measurement of the diameter of the stellar image on the photographic plate and the use of an empirical formula connecting diameter of image with star magnitude.

(ii) By visual comparison of the star's image with a series of artificial star images varying progressively in magnitude.

(iii) By measurement of the density of the out-of-focus image of the star.

(iv) By measurement of the total reduction in light produced by the focal image of the star when placed immediately over a pin-hole only slightly larger than its diameter.

(v) By placing a coarse grating on the object-glass of the telescope so that a series of minute spectra of each star are formed. The relative brightness of the spectra of various orders can be calculated.

* See Ross, *Astrophys. Journ.*, Dec. 1922, and other papers in this and similar publications.

Whatever method be adopted, certain standard comparison stars are photographed also, in order to eliminate atmospheric effects. Thus the measurement of star magnitudes largely resolves into the comparison between the images of two stars of nearly equal magnitude. This, of course, very greatly simplifies the problem and increases the possible accuracy. Other difficulties, however, arise. For example, owing to the unsteadiness of the atmosphere the star's image is not stationary on one portion of the plate, but oscillates about over a small range. Thus any minute portion of the plate may have a somewhat intermittent exposure, and a larger area of plate will be exposed for a shorter time than if the image were perfectly stationary. Various errors obviously arise from such a cause, owing to the peculiar characteristics of the photographic plate.

III

PHOTOMETERS OR INSTRUMENTS FOR MEASURING DENSITY

MANY forms of photometer have been designed which are set by adjusting the brightness of two parts of an illuminated field until they appear equally bright to the eye. By careful design a good observer can read to about 0·005 to 0·002 in density, but this accuracy requires practice, and such measurement is very tiring to the eye if continued for a long time.

Recently, photometers using photo-electric cells, thermo-elements, or selenium cells have been coming into use. They have the great advantage that no eye estimation is required. Koch * has designed his well-known photometer so that errors due to variations in the illuminating source are avoided by the use of two photo-electric cells, one acting as a constant 'leak'. Moll † has designed an instrument using a thermo-element of very small heat capacity, whereby the density of very small areas of plates can be very quickly measured

* *Ann. d. Phys.*, vol. xxxix (1912), 705.

† *Proc. Phys. Soc.*, June 1921.

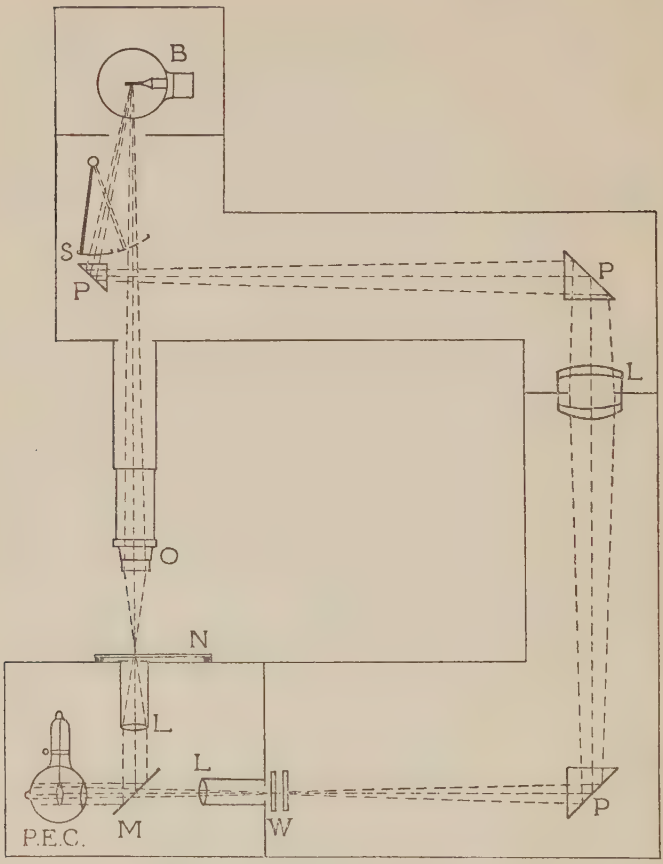

FIG. 6. *Diagram of Photometer.*

L = Lenses.
P.E.C. = Photo-electric Cell.
P = Totally reflecting Prisms.
B = Electric Lamp Bulb.
S = Shutter.
O = Microscope-objective Lens.
M = Semi-silvered Mirror.
W = Wedges.
N = Negative being measured.

and recorded. The illuminating source must be kept constant. Professor Sampson, Mr. Baker * and their associates at Edinburgh have designed a similar instrument for measuring spectra, in which a photo-electric cell and a special type of electrometer are used, so that variations in the voltage applied to the cell are eliminated.

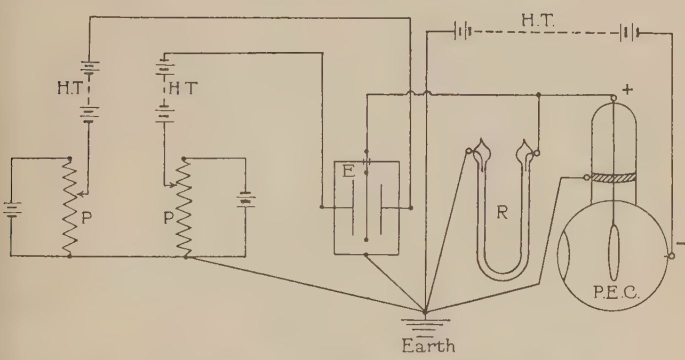

FIG. 7. *Diagram of Connexions for Photometer.*
H.T. = High-tension Batteries.
P.E.C. = Photo-electric Cell.
R = Xylene Alcohol Resistance approx. 10^9 ohms.
E = Electrometer.
P = Potentiometers.

Recently we have tried to obtain a photometer† in which both variations in the source of illumination and in the voltage applied to the photo-electric cell are eliminated, and also where all densities can be measured with equally good

* E. A. Baker, *Journ. Sci. Inst.*, Aug. 1924, and *Proc. Roy. Soc. Edin.*, vol. xlv, Part II.

† G. M. B. Dobson, *Proc. Roy. Soc.*, vol. 104, 1923.

accuracy, while the values of the density are read off directly in one operation, the density of the background being automatically allowed for. The principle of the method now in use will be seen from Figs. 6 and 7.

Light from any suitable source is divided into two beams, which, after traversing different paths, fall on to the sensitive element. One beam passes through the plate whose density is to be measured, while the other passes through a standard optical wedge, which can be adjusted so that the light is reduced in any known proportion. A shutter allows one or other beam to fall, as desired, on to the sensitive element. When in use the optical wedge is adjusted until there is no change in galvanometer reading when the shutter is changed over.

A 'null' method of working would seem to be ideal, but this is not easy to design, and the present arrangement has very nearly the same advantages, and is extremely simple and inexpensive.

To allow for the difference in the optical transmission coefficients of the two paths, and also to eliminate the density of the 'background' or unexposed part of the photographic plate, a subsidiary wedge is provided close to the standard wedge. Prior to measuring the density of the image, the standard wedge is set in its zero position, and the plate arranged so that light

passes through the unexposed part. The subsidiary wedge is now adjusted until the galvanometer shows that there is a balance. Next the image to be measured is placed under the light, and the standard wedge adjusted until there is again a balance. The reading of the standard wedge then gives directly the density of the image.*

It will be noticed that slow changes in the sensitivity of the cell, e. g. those caused by changes in the applied voltage and slow changes in the illuminating light, are quite unimportant with this method of working. Again, it should be noted that only the extreme deflection of the galvanometer need be visible, and its zero position may be quite outside the field of view. This permits the use of a very high magnification, and thus gives great accuracy.

The beam of light passing through the plate to be measured is focused by the lens (O), so that the image of the short straight filament of an electric lamp—which forms the source of light †—is thrown

* In the British Photographic Research Association's modification of this instrument one wedge only is employed. This is set to give a balance when the unexposed part of the plate is being measured, and the scale against which the wedge readings are made is moved along so that the reading is zero. On re-setting the wedge for the image the density is given directly by the scale reading. The apparatus is thus somewhat simplified.

† If desired, the image of an illuminated slit can be used.

on the plate. A diaphragm immediately behind the plate has a hole in it which is a little larger than the image of the filament. This serves to cut out stray light. In some cases it is desirable to measure the density of a moderately large area, say a few square millimeters, while in others, such as when measuring the density of spectral lines, a very small area must be used. To make the instrument adaptable to any purpose a series of lenses can be used to focus the light. For the smallest areas a microscope objective is employed.

The plate to be measured is carried on a mechanical stage, with scales and verniers for measuring distances along two axes at right angles to one another. In the first method of working (Method A) it is necessary to measure the distance from the zero line on the wedge image to a place on this image where the density has some fixed value. To obtain this, the mechanical stage is adjusted until the spectral line in question comes under the light, and the vernier reading is taken when the light is passing through the zero line. The standard wedge of the photometer having been set to give the fixed density, the mechanical stage is moved until a point on the spectral line is found which has the required density. Reading the vernier again, we obtain by difference the distance we require. This type of setting can be made with very great accuracy, readings being repeated to about

0·05 mm., which is as close as can be read with any certainty on the verniers. If desired, readings may be made at three fixed densities, and the mean taken, thus gaining accuracy.

The actual instrument employed by the writers contains a potassium photo-electric cell as the sensitive element. Since, when the densities of very small areas have to be measured, the amount of light reaching the cell must be small, it is desirable to make the galvanometer as sensitive as possible. We actually use a high-resistance leak of about 10^{10} ohms in series with the cell and high-tension batteries, and a sensitive, quick-period string electrometer * is shunted across the leak. The resistance consists of a mixture of very pure alcohol and xylol in a glass capillary tube with platinum electrodes. Densities can be measured with a probable error of 0·0005, i.e. a tenth of one per cent. of the transmitted light.

We find that these cells work quite satisfactorily if the applied voltage be about five volts below that at which the cell passes a current when not exposed to light. This happens in our particular cell at about 197 volts, and we normally use about 185 to 193 volts.†

* Lindemann and Keeley, *Phil. Mag.*, vol. xlvii, March 1924.

† Objections have frequently been raised to photo-electric cells on the score of short life, fragility, and change of sensitivity. The cell used by the authors is now eight years old

In order to be able to measure both large and small densities, the intensity of the light is made easily variable over a large range either by varying the voltage on the lamp or by an optical wedge which reduces both beams alike. The sensitivity of the whole apparatus is adjusted so that a conveniently large deflection of the galvanometer is produced by a very small light when only small densities are being measured. Then when large densities have to be measured the light is increased so that the same deflection is still obtained. Thus all densities can be measured with equal accuracy.*

and is still in perfect working order, although no special precautions have been taken. As regards change of sensitivity, even if there is a variation, it can have no effect with this type of photometer. There are grave objections to the use in a photometer of thermo-elements or bolometers which indicate total energy received, since it is much easier to eliminate stray *light* than stray *heat* which may be emitted from all parts of the apparatus.

The cell used has the surface of the alkali metal converted into the hydride, and is filled with argon at a low pressure. This type of cell is very much more sensitive than the high-vacuum cell with pure metal, and is perfectly satisfactory for use in photometers.

* If an instrument were required to give the very greatest possible accuracy (say less than 0·1 per cent.) it would be advisable to make the two paths of the light which reaches the cell exactly alike, using similar lenses, &c., in each beam, and making the patches of light falling on the cell identical in size and position, thus eliminating the possibility of fatigue effects.

It should be noticed that the method of allowing for the density of the 'background' or unexposed parts by subtraction is not strictly accurate, since the amount of developer fog will probably be less in the image than in other parts, as the bromide formed during development will reduce the amount of fog (see Ch. V, § 10).

A modification of this instrument has been brought out by the British Photographic Research Association, using a selenium cell, but the principle is the same.*

When the photometer is used to measure the densities of spectral lines the area of the plate which is illuminated must be narrower than the width of the lines. There is, however, a lower limit to the size of the image which can be used; this is due, not to the photometer, but to the photographic plate. Thus, in one case, when measurements were being made on spectral lines taken through a wedge, an area $0{\cdot}2 \times 0{\cdot}02$ mm. of the plate was illuminated by throwing an image of a small straight filament on to the plate by means of a $\frac{2}{3}$-inch microscope objective. Measurements with this lens were quite useless, as adjacent minute areas of plate might vary 5 per cent. to 10 per cent. in density, and quite irregular results were obtained, even on Process Plates. It is, of course, obvious that when the area which one is measuring contains only a few

* Toy and Rawling, *Journ. Sci. Inst.*, Sept. 1924.

silver grains, irregular results must occur, but one might have expected that an image of the size used would contain sufficient grains to give uniform results. On substituting a 2-inch microscope objective which gave an image $0{\cdot}6 \times 0{\cdot}06$ mm. good results were obtained, even with special rapid plates. We thus see that if spectral lines photographed through a wedge are to be measured, a moderately wide spectrograph slit must be used, giving lines approximately 0·1 mm. wide on the plate, i. e. about twice the width of area illuminated when measured on the photometer. Also, if uniform spectral lines are to be measured, i. e. lines not taken through a wedge, and a narrow slit must be used, then a very long and narrow image must be employed in order to obtain the necessary illuminated area of plate. It will require considerable care to insure that such an image falls exactly along, and in the centre of, the spectral lines.

It will be seen that the coarse, granulated nature of the plate must cause similar errors in any method of measuring the density of small areas. Thus, if the sharp outline method, described in Ch. II, § 6, be employed, it will be subject to the same type of error if fine spectral lines be used, though it will not be immediately revealed, as it is when the measurements are made with a photometer.

In the photometer designed by Mr. E. A. Baker

a transparent mirror reflects a portion of the light passing through the photographic plate being measured into a microscope. This has the great advantage that the operator can see a magnified image of the actual area of plate which he is measuring and detect stray dust specks or pin-holes.

NOTE.—The staff of the British Scientific Instrument Research Association have pointed out that, for the highest accuracy, it is desirable to ensure that the light always passes through exactly the same part of the window of the photo-electric cell, or to use a cell with a flat window, otherwise varying reflection may cause errors.

IV

THEORETICAL BEST CONDITIONS OF WORKING

We shall in this chapter examine various details in the photographic process in order to find the best conditions for obtaining accuracy in photometric measurements. In general it is by no means necessary to adhere exactly to these conditions, and in certain cases convenience or other considerations justify one in departing more or less from the optimum conditions here found.

1. *Optimum Density of Image.*

Any errors introduced by a change of γ from one part of the plate to another, such as by uneven development causing the plate to be more fully developed (and therefore to have a larger value of γ) in one part than in another part, will be less if the exposures given are relatively small, so that the final densities obtained are in the neighbourhood of 0·5.

Thus let Fig. 8 represent two characteristic curves for two parts of a plate having slightly different amounts of development, one part con-

taining the standard image, the other part containing the experimental image. Suppose we are using Method A (1) and let curve *A* correspond to the development given to the standard image, while curve *B* corresponds to that of the experimental image. If we choose a high standard density—say d_1, then *J*, corre-

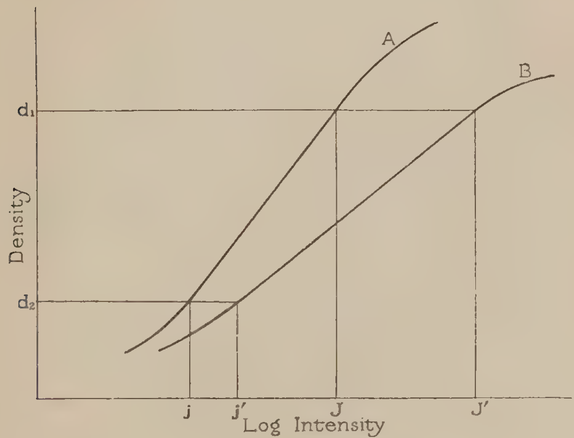

FIG. 8.

sponding to the point *J*, is the intensity of light producing this image, and, if the amount of development had been everywhere the same, all images where the density was d_1 would have received an amount of light *J*. If, however, the experimental image has received a different development (curve *B*), the part of the image whose density is d_1 will have received an exposure *J′*,

corresponding to the point J', thus introducing an error given by JJ'.

If, however, we had chosen a small value for the standard density, say d_2, the corresponding error would only have been that given by jj', a much smaller percentage error.

Increasing the amount of development may also alter the value of the inertia (i), but the effect of this, if we could assume γ constant, is to move the curves parallel to themselves without changing their slope, and the same error will be introduced whatever density be employed.

Again, possible slight differences in the character of the photographic plate from one part to another may change either γ or i and the same arguments hold.

It must be noted that the small density must be obtained by a small exposure, not by short development. As shown later, development should always be as long as possible, provided no appreciable chemical fog is produced. There is also a minimum limit to the density which should be used, since at small densities the curve has a much smaller slope, and it will be shown that the maximum slope should be used. We thus find that it is desirable to use a density near the lower end of the straight portion of the curve. In general a density of about 0·3 to 0·5 will be satisfactory.

In addition to the above, there are two further

advantages in using small densities, as (1) in many photometers greater accuracy can be obtained with small densities, since more light is available, and (2) errors due to uneven development owing to insufficient stirring will be less with small densities, since less soluble bromide is formed to restrain development.*

2. *Optimum Length of Development.*

A maximum length of development is fairly definitely fixed by the first appearance of appreciable developer fog. Except for this, a long development is an advantage, since (1) we obtain a larger value of γ and therefore greater accuracy in estimating the incident light (see § 3), and (2) since D and γ increase with the length of development (t) approximately as $(1 - e^{-kt})$, slight differences in the effective time (t), due to uneven development, will produce less differences in D or γ if t be large.

Thus we find the rule that development should be as long as possible, providing no appreciable chemical fog is produced.

3. *Effect of the Value of γ on Accuracy.*

If we assume the relation between the density and the exposure to be given by the usual equation

* See foot-note, Ch. V, § 5.

$$D = \gamma \log It^p - i,$$

then
$$dD = \gamma \frac{dI}{I},$$

or
$$\frac{dI}{I} = \frac{dD}{\gamma}.$$

Hence the percentage error in I produced by a small error in the measurement of D varies inversely as γ. If the density of the image lies on a part of the characteristic curve outside the straight central portion, the effective value of γ, i.e. $\frac{\partial D}{\partial \log I}$ is smaller, and the accuracy is diminished.

4. *Relative Advantages of Different Positions of the Wedge in Spectroscopic Work.*

It has been stated in Ch. II, § 2, that in spectroscopic work the wedge may be placed either at the slit of the instrument or in front of the photographic plate.* The following considerations would appear to indicate that in order to minimize

* If the wedge were placed in any other position (as, for example, in contact with the lens which focuses the light on the plate) it is obvious, from simple geometrical considerations, that as any particular point of the plate is receiving light from the whole surface of the lens, and as the light from separate portions of the lens is being reduced by the wedge in different proportions, the light incident on any point of the plate has been reduced in intensity by an amount which is a complicated function of the effective wedge constant.

errors due to scattered light the wedge should be placed immediately in front of the plate.

We will suppose that we are dealing with a beam of monochromatic light.

Let l be the length of the slit, d its width, A the amount of light falling on it per unit area, m the magnification of the spectrograph. If K is the density gradient of the wedge, the ratio of the incident light I_o to the transmitted light I at a point distant y from the base line of the wedge is given by the equation

$$I = I_o \, . \, 10^{-Ky}.$$

We will assume in the first instance that the wedge is in contact with the plate. The direct light L falling on the plate per square centimetre at a distance y from the base line is

$$L = \frac{A \, . \, 10^{-Ky}}{m^2}.$$

The scattered light falling on the same small area is

$$S = Aldb \, 10^{-Ky},$$

where b denotes the fraction of the incident light which is scattered in the instrument and which would fall on unit area of the plate if the wedge were removed.

Hence $$\frac{S}{L} = ldbm^2.$$

When the wedge is placed immediately in front of the slit, let its constant be K', such that

$K' = mK$, so that the effective wedge constant for measurements made on the plate is the same in both cases. We obtain in a similar way for the amount of direct light L' falling on the plate at a point distant y from the reference line

$$L' = \frac{A \, . \, 10^{-K'y/m}}{m^2}.$$

Similarly for the scattered light

$$S' = aAldb,$$

a being the ratio of the total light transmitted by the wedge to the total incident light, so that

$$aAld = \int_0^l d \, . \, A \, 10^{-K'z} \, dz,$$

or $$al = (1 - 10^{-K'l}) \div K' \log_e 10.$$

In this case

$$\frac{S'}{L'} = \frac{bdm^2 \log_{10} e \, . \, 10^{K'y/m} \, (1 - 10^{-K'l})}{K'}.$$

The figures in Table I show the variation of $S'/L' \div S/L$ for different values of c and Kh, h being the total height of the image of the slit, i.e. $h = ml$ and $c = y/h$. When the wedge is in front of the plate, the ratio of scattered to direct light is independent of the ordinate of the point at which the plate is measured; when the wedge is in front of the slit, this ratio increases rapidly with increasing ordinate.

TABLE I

Values of $S'/L' \div S/L$.

	$c = 0 \cdot 1$	0·3	0·5	0·75	1·00
$Kh = 2$	0·34	0·86	2·15	6·80	21·5
3	0·29	1·15	4·57	25·7	144
4	0·27	1·72	10·9	109	1086
5	0·27	2·75	27·5	489	8686

Since, for accuracy, the wedge will naturally be such that all values of c occur in practice, it is evident that there is great advantage in using the wedge near the plate [see, however, foot-note to Ch. V, § 5].

V

ERRORS AND PRECAUTIONS

1. *Development of Plates.*

It is of the utmost importance that the development should be as uniform as possible over the whole plate, and that it should be well under control, so that all plates can be given a nearly similar development. For this purpose tank development is better than development in a dish, since the temperature can more easily be kept constant.

Various methods have been proposed with the object of securing uniform development over the whole plate. Messrs. Ilford have recommended placing the plates in a recess in the false bottom of a developing dish which is very much larger than the plate. The surface of the plate is then flush with its surroundings, and forms a small part of a large flat surface. A relatively small quantity of developer is poured into the dish, which is then rocked quite slowly, allowing the whole of the developer to flow right across the plate each time. Such a method is very much superior to the common method of placing the plate in a dish

of its own size and rocking gently, which gives excessively uneven development.

The false bottom method referred to above is, however, not free from objection. The products of development are not swept away as quickly as necessary, and the central parts of a uniformly exposed patch are considerably less dense than the edges. To overcome this difficulty, the unexposed parts of the plate have sometimes been specially exposed afterwards, so that when developed they shall have approximately the same density as the images to be measured. We shall show, however, that such additional precautions are unnecessary if suitable methods of developing are used. Further, any slight frilling at the edge of the film appears to cause additional turbulence in the flow of developer, which results in increased development, causing dark marks across the plate. A uniformly exposed plate developed by this method shows a regular decrease in density from edges to centre. It may, however, be pointed out that if a series of small images have to be compared, and these are arranged in two rows symmetrically placed on the plate and parallel, say, to the long axis of the plate, then, if rocking be chiefly about this long axis, the errors from uneven development are not large.

Mr. O. Bloch has recommended covering an ordinary 'squeegee' roller with soft velvet, the whole of which is soaked in developer, and rolling

the plate in various directions with it. The object of this is to remove the layer of developer lying immediately against the surface of the plate, and continually to supply fresh developer to the plate. The method appears to be a distinct improvement on the other dish methods. Dr. Clark, of the British Photographic Research Association, has obtained very good results by brushing the plates continually with a wide, soft camel's hair brush during development. This serves a similar purpose to Mr. Bloch's roller.

In the above methods it is somewhat difficult to control the temperature, if it is necessary to develop all plates as nearly equally as possible.

We have tried developing the plates in an ordinary developing tank, keeping the plates very wide apart, and only half filling the tank with developer. During development the tank was kept violently shaken about and continually reversed. The splashing of developer should remove the products of development very quickly from the surface of the plate. The method was not as satisfactory as was expected, and uniformly exposed images still tended to be denser at their edges than at the centre. Further, most tanks suitable for this method are made of metal, and stray markings are apt to appear on the plate due to contact of the metal with the developer.

Finally, we have adopted another method of development. It appears that with most methods

the products of development cannot be removed so quickly from the surface of the plate that their concentration is negligible, i. e. the stirring is not sufficient. If very violent eddies could be produced close to the film, uniformly over the whole plate, presumably more uniform development would result. To obtain this condition, we made a glass developing tank in which the plates are held in grooves against the vertical walls. A plunger of glass or ebonite, nearly fitting the inside of the tank, can be moved up and down within it. There is a clearance of about a millimetre between the edges of the plunger and the surfaces of the plates. As the plunger is moved the developer is forced at high velocity between its edges and the plates. Violent eddies are thus produced right against the plates. Above and below the plates are pieces of glass of the same thickness, so that there is no appreciable discontinuity at the top and bottom edges of the plates. The plunger travels a few centimetres beyond the plates, so that the whole is treated uniformly. The velocity of the plunger is kept constant over the whole of its travel. The plunger may be worked by hand or by a heart-shaped cam driven by a motor, the velocity in each case being kept as nearly as possible constant. The plunger travels at a speed of some 20 cm. per sec., the developer attaining a speed of about 2 metres per second past the plates.

This method of developing has been tested by (1) exposing plates uniformly all over, and measuring the resulting density at all parts, (2) exposing uniformly twelve small patches distributed over the plate, (3) exposing some large and some small patches uniformly, and measuring the resulting density. These tests show that this method of development gives much more uniform results than those obtained in the other methods tried, and that a large part of any remaining irregularities is due to unevenness in the coating of the film on the plate. Figs. (9) and (10) show the density distribution over two commercial plates (12 × 9 cm.), uniformly exposed all over. The one was very carefully developed in a dish with false bottom, and the other in the special tank described above.

2. *Washing and Drying of Plates.*

Plates which are sensitive to green, yellow, and red light are sensitized to these colours by bathing in a solution of a special dye. When such a plate is washed for a short time only, it will often be noticed that the clear parts are still slightly coloured. It will be shown later that it is important that the image, if measured in a photo-electric photometer, should be grey. The slight colour remaining from the sensitizing dye may cause some small error, particularly as the

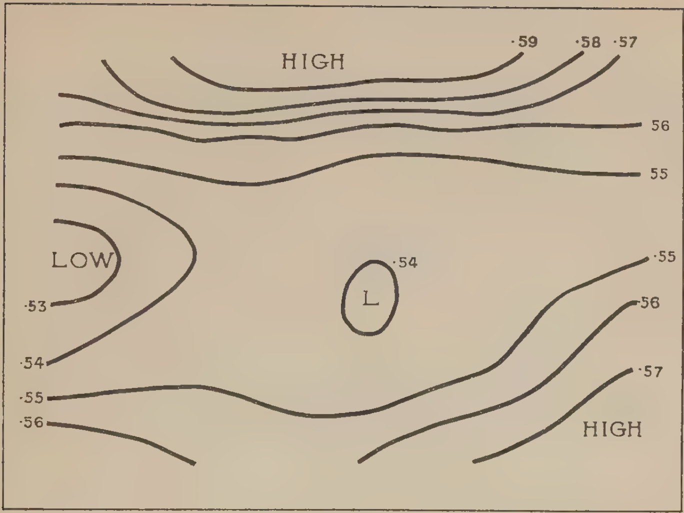

FIG. 9.
Developed in dish with false bottom.

FIG. 10.
Developed in Special Tank.

remaining dye will probably be unevenly distributed over the plate. If the plates are washed for a long time, say twenty-four hours, practically all trace of colour disappears.

If, after washing, one part of the plate be dried rapidly while another part be dried slowly, small differences in the density of the image can often be observed, which correspond to the different rates of drying. Probably the best method of drying is to shake all surplus water off the plate, and then to dry it in the open or in a good current of air.

3. *Uniformity of Plates.*

Having reduced the errors of development as far as possible by the method first described, we are still left with those due to unevenness in the plate itself. These variations in the plate may be due to (1) a change in the thickness of the film, or (2) change in the character of the film such as might be caused, for example, by uneven drying of the plate when manufactured. The measurement of the variation in the thickness of the film *in situ* is not easy, since the whole film is generally only some 15 μ thick. We have found it possible to obtain fair results by measuring the optical thickness with the aid of a Michelson interferometer, in the case of film coated on 'patent plate' glass.

With commercial plates developed in the special

tank described, the main variations in density seem to be accounted for by variations in the thickness of the film. Greater uniformity of film can be obtained if the plates are made of flat glass such as 'patent plate', and the plate makers are usually willing to supply plates on this glass at a special price.

Small plates are cut from a larger plate in which the emulsion is often much less regular round the edge than in the centre. It is desirable to arrange that these edges are cut off before the smaller plates are cut from the parent plate.

In the case of images whose density is less than about 1·0 most of the silver grains forming the image are situated in the upper part of the film. In this case a small change in the thickness of the film will not greatly affect the density of the image. If, however, the image were so dense that the silver grains forming it were distributed through the whole thickness of the film, then, obviously, a thin part of the film must have too light an image. For this reason it seems preferable to use plates covered with a thick layer of emulsion, and to measure the images where the densities are fairly small. Other reasons for keeping the density small are discussed in Ch. IV, § 1.

4. *Uniform Illumination of the Wedge.*

It is of the utmost importance that the light illuminating the wedge should be uniform all along

it, and careful tests should invariably be made to see that this is so. By far the best way is to remove the wedge and make an exposure without it, when the resulting image should be uniform in density in the direction of the gradient of the wedge. The image should be examined with a photometer, since the eye is very insensitive to slight changes of density if these changes are gradual.

It has been suggested that non-uniformity of illumination may be detected by reversing the wedge, and that its effect may be eliminated by taking means of observations with the wedge erect and reversed.* When the illumination varies irregularly along the wedge, as will frequently be the case, this cannot be true.

It not infrequently happens that the mounts for the optical parts cut down the effective aperture at the top and bottom of the lines at each end of the spectrum. Again, if the slit be not exactly uniform in width, the image will not be evenly illuminated. This error is reduced by using as wide a slit as possible.

5. *Elimination of Scattered Light.*

Scattered light is a frequent source of great trouble. Every lens or prism surface scatters a small amount of light, and, in addition, acts as

* Plaskett, *Publications of Dominion Astrophysical Observatory*, Victoria, B.C., vol. ii, No. 12, p. 219.

a mirror, so that a number of faint images are formed at various places along the optical system. By placing the eye in the position usually occupied by the plate it is generally possible to find exactly where these images are formed, and a small black stop fixed in this position, and of such a size that it just covers the image, will cut out this stray light without greatly reducing the main beam. In a good instrument stray light due to reflection from the sides of the tubes holding the optical parts is generally cut out by suitably placed diaphragms, but tests should be made to see whether this source of stray light is present. Again, the surface of the plate scatters a large amount of light, and if the inside of the camera is not well blackened some of this light may be reflected back on to other parts of the plate.*

In the case of a spectrograph, probably the simplest method of testing for stray light is to cover a small part of the centre of the slit, and then give a long exposure, when the image of the spectrum should be crossed by a relatively clear

* When the wedge is close to the plate, some of the light scattered by the emulsion will be reflected back by the wedge on to other parts of the plate. Where parts of the image are very intense compared with other parts, it may be advisable to arrange a black mask before the wedge to cut out the densest parts of the image, without interfering with the parts of the image where measurements are made. This also has the advantage that development is made more uniform since there are no adjacent dense patches.

band. Owing to the effect of 'inertia' in a photographic plate, it is necessary to make a very long exposure or to fog the plate uniformly all over, so as to bring the image on the straight part of the characteristic curve, as otherwise a small amount of stray light may not show up in this test, but may yet cause error in the wedge images.

When one part of the spectrum is very bright and another part weak, it is often impossible to eliminate stray light satisfactorily, the image in the weak part being fogged all over by scattered light from the bright part. In this case the intensity of the bright part must be reduced by means of suitable filters or otherwise. Thus, it is quite impossible to get any accuracy in measuring the ultra-violet end of the solar spectrum, beyond about 3,600 Å, unless the brightness of the visible part be enormously reduced. It must be remembered that short waves are scattered very much more by glass surfaces than are long waves.

6. *Measurement of Density of Non-grey Materials.*

Photo-electric cells containing one of the alkali metals or its hydride as the sensitive element do not respond uniformly to light of all colours, but while slightly sensitive over the whole visible and ultraviolet spectrum, have a very marked maximum of

sensitivity in a certain region. The wave-lengths of the maximum sensitivity of the metals and their hydrides are approximately as follows: *

TABLE II

Metal.	*Wave-length.*
Sodium	0·30 μ
Potassium	0·44 μ
Rubidium	0·47 μ
Caesium	0·52 μ

Consequently, if the medium whose density is being measured be slightly coloured, different values of the density will be found with the different types of cell. Photometers using a thermo-couple or selenium cell with an electric filament lamp as source of radiation are in an even worse case, since the effective wave-length used is then in the infra-red, and many materials which are 'grey' throughout the visible region have a very different absorption coefficient in the infra-red. Another point to be noticed is, that if two filters of densities, say, d_1 and d_2 be combined together, the density of the combination is given by

$$D = d_1 + d_2,$$

so long as the absorption coefficent is constant over the range of wave-lengths used in measuring the density, but this is no longer true if the absorp-

* Somewhat different figures have been obtained by different investigators, due presumably to their methods of preparing the cells. See *Astrophys. Journ.*, 1920–5.

tion coefficient changes with wave-length, and numerous troubles may arise in certain classes of work.

7. *The Standard Light.*

We have shown how, whatever method of photometry be used, the results of different plates may be made comparable by the use of a standardizing image on each plate. It is now necessary to consider the best means of producing this standard image. In many cases it is important that the light should be exactly uniform over a considerable area of plate, as, for example, when the image of a wedge is photographed on the plate. We have found it convenient to use a gas-filled electric lamp which illuminates a white matt screen. The plate is exposed to light scattered by this screen, but not to direct light from the lamp. An electrically operated shutter is placed between the lamp and the white screen, together with any colour filter that may be used. The total light from a gas-filled electric lamp varies roughly as the fourth power of the voltage when run at its normal voltage. At lower voltages the index is even greater, going up to about five at half the normal voltage and to about six at a quarter the normal voltage. Therefore it is important to adjust the voltage very carefully. It seems desirable to run

the lamp a little below its normal voltage in order to avoid blackening of the bulb and change in the filament. As a further precaution, the current passing through the lamp at the standard voltage adopted may also be measured, but it should be remembered that, owing to the change of resistance of the filament with temperature, a small percentage change in the current causes a greater change in light than that caused by the same percentage change in voltage (see note, p. 94).

8. *Timing of Exposures.*

Where exposures are of the order of a minute or more, an accuracy of less than 1 per cent. ought to be attained by careful use of hand-operated shutters and a stop watch. In many cases, however, much shorter exposures will be necessary, and an electrical control is greatly preferable. A very simple electrical device, which allows any desired exposure to be given with great accuracy, can be easily arranged. Any simple clock or electrically driven pendulum making second contacts may be used. The shutter, which may be either of the setting or automatic type, is fitted with an electro-magnet and armature, so that it opens at one impulse and closes on the next. A press switch is put in series with the pendulum contact and shutter, so that by pressing the switch just before one pendulum contact the shutter will

be opened. The shutter then remains open for the desired number of seconds, and, when the switch is pressed again, the shutter will close at the next pendulum contact. If strong springs be used on the shutters, so that they open and close rapidly, an exposure of any exact number of whole seconds may be made with very great accuracy.

9. *Position of Standard Images on the Plate.*

We have seen that by taking suitable precautions the differences of density over a uniformly exposed plate should be small. To eliminate the residual differences of density, Professor Lindemann suggested to the writers that the plate should be exposed to the experimental light through a grid or chess-board mask. This grid should then be shifted one bar width and exposed to the standard light. Thus small areas exposed to the standard and experimental lights would be adjacent to each other all over the plate, and errors due to local changes in plate sensitivity or development would be avoided.

In a paper just published,* Mr. L. A. Jones has used a similar method. He exposes the plate through a grid and the intensity of radiation is made to decrease logarithmically along the grid. When exposed to the standard light the direction

* Jones, *Journ. Opt. Sci. America*, May 1925.

of gradation of the light is reversed. The use of a reversed logarithmically graded standard light increases the possible range in intensity.

10. *Allowance for Background Fog.*

As previously stated,* the density of a photographic image is usually taken to be the difference between the density as measured through the image and that as measured through an unexposed part of the plate. This is, of course, correct if the density due to 'fog' be the same all over the plate, and independent of the presence of any image due to light. One can see at once that this cannot be strictly true, and that such an assumption will be more in error the larger the density of the light image; for (1) where there is a dense image there will be a greater concentration of soluble bromides in the film during the development, which will restrain the production of chemical fog. (This effect will be minimized by thorough stirring during development).† (2) If a large number of silver halide grains have been acted

* See Ch. I, § 2.

† Mr. Bloch suggests using minute clear areas in the centre of the images for the measurement of fog. The condition of the developer will be roughly the same for these small areas as for the adjacent image, but it has been pointed out that such areas are affected by irradiation from surrounding portions, and therefore give too large a value for the fogging (see *Faraday Soc. Symposium*, 1923).

on by light and so form the light image, there will be fewer grains available to form fog. This is an additional reason for avoiding very dense images.

The only real solution of the difficulty is to avoid any appreciable chemical fog, so that the density of the background to be subtracted from the density as measured through the image is almost negligible.

If in measuring the density of the background its density is not absolutely constant all over the plate, we may either take the average of the background for the plate, or we may measure the background density near each image and subtract this value from the measured density of the particular image. The latter method appears to be the most accurate.

11. *The Effect of Width of Slit and Structure of Spectrum Lines.*

The structure of spectrum lines and the optimum width of slit for general spectroscopic work are discussed in text-books of optics,* and we shall here only mention a few special points which are important in photographic photometry.

The radiation emitted by a so-called line source is never strictly monochromatic, but extends over a small range of wave-lengths. In some cases

* e.g. Schuster and Nicholson, *Theory of Optics*.

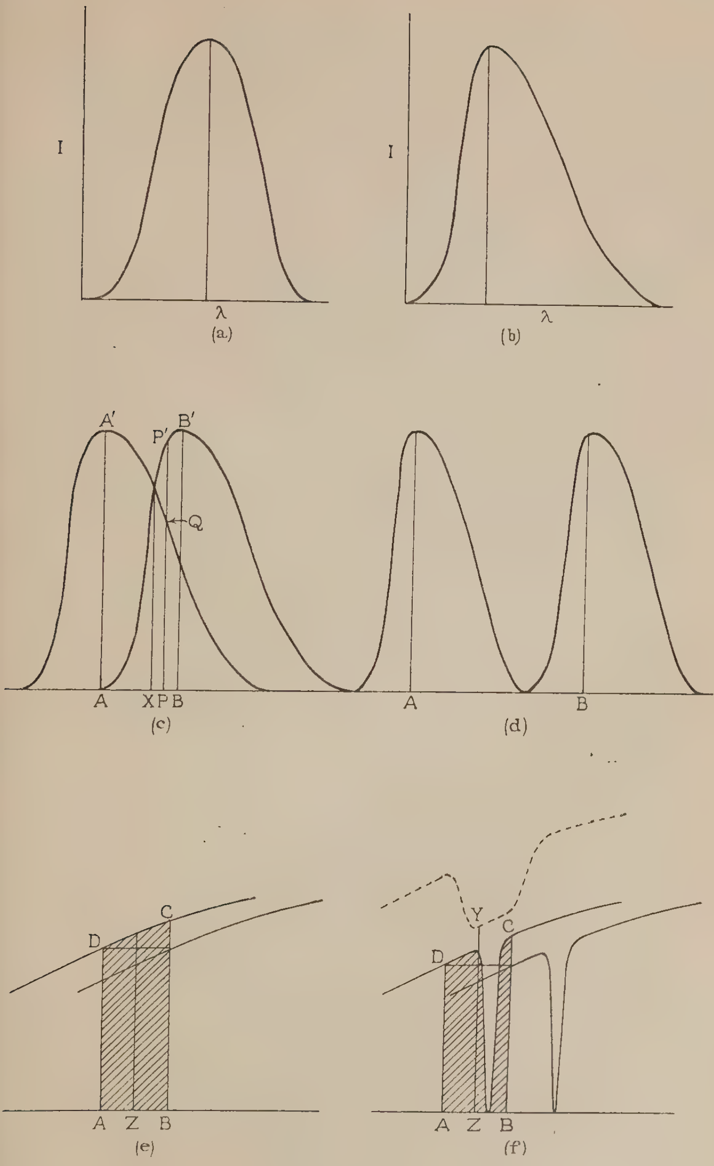

FIG. 11.

the distribution of energy in the line is symmetrical, as shown in Fig. 11 (*a*), and in other cases it is unsymmetrical, as in Fig. 11 (*b*). The wave-length is generally taken as that corresponding to the maximum intensity of radiation. This effect and diffraction within the spectrograph combine to produce an image of the slit which differs from its geometrical image. Figs. 11 (*a*) and (*b*) correspond to infinitely narrow slits without diffraction effects. The images produced by slits of finite width can be regarded as the resultant images corresponding to a number of infinitely narrow adjacent slits. If AB, Fig. 11 (*c*), is the width of the geometrical image of the slit, the energy distribution at any point P will be given by the sum of the ordinates at P for all the elementary curves, and will be proportional to the sum of the areas $AA'QP$ and $BB'P'P$.

This has a maximum value at X, corresponding to the point of intersection of the two extreme curves, so that the position of maximum intensity in an unsymmetrical line depends on the width of the slit. If lines of different asymmetry be present, their maxima will be moved by different amounts as the slit is widened, and this may produce an error in apparent wave-length.* As the width of the slit is increased, there is a gain in intensity up to the point indicated by Fig. 11 (*d*),

* Merton and Harrison, *Proc. Roy. Soc.*, A, 1922, vol. 101, p. 431.

AB representing the slit-width, after which any further increase only broadens the line.

Since, in photometric work, density of the lines has to be measured, the slit must be of sufficient width to give a portion in the centre of each line of uniform density, which must be sufficiently large to be measured in the photometer. Moreover, as shown in Ch. III, paragraph 4, irregularities in the grain of the plate cause appreciable errors if the illuminated area is too small. There is an upper limit to the width of the slit which can be used, as increasing the slit-width tends to make adjacent lines run together with consequent loss of resolving power. The best width of slit must in each case be decided by trial, regard being paid to the above considerations. It will be seen that it is an advantage to use a plate with as fine grain as is consistent with the requisite speed. In general, the size of grain increases with the speed.

Continuous Spectra. In a continuous spectrum the intensity for any slit-width may be found as before, but now the range of wave-lengths is almost infinitely great. If the distance *AB*, Fig. 11 (*e*), is equal to the width of the geometrical image of the slit for a definite wave-length λ, and the two intensity curves drawn correspond to the two edges of the slit, the intensity at any point is represented by the area *ABCD*, while the wave-length, as deduced from a symmetrical line in a

comparison spectrum, is given by the point Z half-way between A and B. Thus the intensity is proportional to $ABCD$.

If absorption lines are present, they will be broadened and filled up by the overlapping of the spectra (Fig. 11 (f)). The intensity is represented, as in the previous figure, by the area $ABCD$, and it follows from geometrical considerations that the width of the absorption line is increased and that the minimum intensity is not zero, even though it may be zero for an infinitely narrow slit. The dotted line shows the intensity curve for a width of slit corresponding to AB, the ordinate ZY being proportional to the shaded area $ABCD$.

Since the dispersion usually changes with wave-length, allowance for this must be made in the calculations of the distribution of energy both in the continuous and line spectra.

Note to § 7.—In a recent paper (*Journ. Sci. Inst.*, Oct. 1925) the Research staff of the G. E. C. Ltd. discuss the best type of standard lamp. They find the vacuum type preferable and show that special attention must be paid to the fixing of the filament contacts.

VI

PLATES AND DEVELOPERS

1. *Plates.*

In the choice of the best photographic plates for any particular problem, regard must be paid to the following characteristics of the plate:

(*a*) Speed.
(*b*) Range of colour sensitivity.
(*c*) Power of reproducing fine detail.
(*d*) Freedom from chemical fog.
(*e*) Contrast.

(*a*) **Speed.** The slowest plate should be chosen which, with the amount of light available, will give rise to a measurable image. The faster the plate, the more trouble is likely to be experienced from chemical fog. As the speed of the plate increases, the size of the grain often increases also, and this increase in grain is detrimental to the production of fine detail (see § *c*).

(*b*) **Range of colour sensitivity.** In spectrographic work we are generally concerned with a definite range of the spectrum, and the plate must be sensitive over this particular region. The so-called 'ordinary' photographic plate is

sensitive in the range 0·45 μ to 0·2 μ, the 'isochromatic' between 0·5 μ and 0·2 μ, the 'panchromatic' between 0·7 μ and 0·2 μ with a somewhat less sensitive portion in the green.

Panchromatic plates should be put into the plateholders, developed and fixed in absolute darkness. As in all accurate photographic work, development must be carried out by the time and temperature method, the length of development being limited by the appearance of chemical fog. This time of development must be found from a few trial plates.

(*c*) **The power of reproducing fine detail.** For the reproduction of fine detail, a process plate is essential. The ordinary process plate is insensitive to the green, yellow, and red, and is very slow. Panchromatic process plates can be obtained, sensitive to the whole visible spectrum and considerably faster than the ordinary process plate. It should be remembered that the power of reproduction of fine detail usually diminishes with increasing plate speed.

(*d*) **Fog.** The presence of fog on a plate which has been exposed and developed with the greatest care may be due either to the developer or to the plate or to a combination of the two. Certain types of rapid plates, even when developed in accordance with the formula of the manufacturer, show signs of fogging which render them useless for exact quantitative work.

If the fog is not very apparent, then, as indicated in Ch. V, § 10, a correction may be made, which is not permissible when the fog is at all appreciable.

(*e*) **Contrast.** For strong contrast a hard process plate should be used if sufficient light is available.

2. *Developers.*

For use in Method A, a hard developer, giving great contrast, is most suitable, while in Method B we generally require a soft developer. In any case, it is important that the developer should be free from any staining action which may produce a coloured effect. Of the hard developers, alkaline hydroquinone may be cited as typical. As regards soft developers, we have found developers of the paramido-phenol type to be quite satisfactory. At first sight it might be supposed that long time of development with weak developer would give the most satisfactory results. It is, however, found that strong developer gives greater uniformity, and one should, as far as possible, adhere to the manufacturer's formula. A suitable temperature is from 50° F. to 65° F., and in any case development should not be carried out at a temperature above 70° F., since with increasing temperature the rate of production of fog increases faster than the rate of development of the true image.

VII

EXAMPLES OF PHOTOMETRIC MEASUREMENTS.

In this closing chapter we give a brief account of three diverse problems in which photographic photometry has been employed by the writers. The object of doing so is not to show the accuracy which can be obtained, since in many cases the work was carried out some time ago, before several sources of error had been eliminated. The purpose is rather to illustrate by a few practical examples the methods which have been given in abstract above, and the limitation of the problems described to those on which the authors have worked is for convenience in supplying detail.

1. *Temperature Measurements.*

In an investigation of the temperature of the crater of an electric arc between carbon electrodes in a gas at high pressure, it was necessary to determine the intensity distribution in the spectrum of the light emitted by the crater during short

intervals of time. Such an arc is difficult to maintain in a steady state, so that methods involving visual comparison of intensities in different regions of the spectrum are not feasible. The light from this arc is so intense that a photographic record of it is easily obtainable in the short time during which it is burning steadily. A spectrograph combined with a neutral wedge, as described in Chap. II, § 2, enables the crater spectrum to be registered at any instant at which it is judged to be brightest, and consequently at its maximum temperature under the given conditions.

In addition to a wedge spectrum of the crater of the arc under high pressure, a similar spectrum of the crater of the arc under atmospheric pressure is photographed on the same plate. As is shown later, measurement of the heights of the two spectra at corresponding points enable us to determine the relative intensities in the two sources, and from the ratio of these intensities we can derive a relation between the temperatures of the sources.

General Description. The light from the crater is arranged to fall on the slit of the spectrograph, so that its intensity in a direction parallel to the slit is uniform. The source of light is at the focus of an achromatic lens of about twelve inches focal length, and after refraction through this lens falls on a cylindrical lens which condenses the beam in a direction perpendicular to the length of the slit.

In order to keep the light accurately on the slit during an exposure, it was found convenient to interpose, between the source and the convex lens, two totally reflecting prisms which could be rotated, one about a horizontal axis, the other about a vertical axis. In this way any movement of the crater up or down or sideways could be readily compensated. After transmission and dispersion in the spectrograph the light traverses a neutral wedge placed in contact with the photographic plate, and forms on the plate a spectrum (Fig. 12 (*a*)). The curved boundary of this spectrum varies in height, as measured from the base line, the variation depending upon the distribution of light in the source, the absorption in the apparatus including the wedge, the dispersion of the spectrograph prism, and the colour sensitivity of the plate. Every plate is standardized by photographing on it the spectrum of a standard source, in this case an arc in air at atmospheric pressure.

Theory of Method. The energy distribution in the spectrum of an incandescent body at a temperature T is given by the Planck expression

$$E_\lambda \, d\lambda = \frac{A_\lambda}{\lambda^5} \cdot \frac{d\lambda}{e^{\frac{c}{\lambda T}} - 1},$$

where $E_\lambda \, d\lambda$ denotes the energy in the wavelength interval λ to $\lambda + d\lambda$, c is a numerical constant whose value is assumed to be 14,500 micron-degrees, and A_λ is a factor which is constant for

a 'black' or a 'grey' body, but varies with the wave-length for a selectively emitting body. We assume that the carbon arc radiates as a grey body, though the method with certain limitations is applicable to bodies which are neither black nor grey.

If E_λ and S_λ denote intensities in the experimental and standard sources respectively, which are assumed to be at temperatures T and T_s,

then
$$\frac{E_\lambda}{S_\lambda} = \frac{e^{\frac{c}{\lambda T_s}} - 1}{e^{\frac{c}{\lambda T}} - 1}. \quad . \quad . \quad . \quad . \quad (6)$$

If α_λ represents the fraction of the light of wave-length λ transmitted by the instrument, $\alpha_\lambda E_\lambda$ and $\alpha_\lambda S_\lambda$ will represent the light intensities incident on the wedge, and their ratios will be represented by the expression on the right-hand side of equation (6). If y and y_s are the ordinates for a given density in the two spectra, corresponding to wave-length λ, and K_λ the wedge constant for this wave-length, then from equation (1), Chap. I, § 3,

$$\log \alpha_\lambda E_\lambda = K_\lambda y + \log I,$$
$$\log \alpha_\lambda S_\lambda = K_\lambda y_s + \log I,$$

the last term $\log I$ being the same in both equations, as we always measure the wedge spectra of both sources at the same density.

Hence
$$\log \frac{E_\lambda}{S_\lambda} = K_\lambda (y - y_s).$$

Similarly for another wave-length λ'

$$\log \frac{E_\lambda'}{S_\lambda'} = K_\lambda'(y' - y_s'),$$

so that

$$K_\lambda (y - y_s) - K_{\lambda'} (y' - y_s') = \log \left(\frac{E_\lambda}{S_\lambda} \div \frac{E_{\lambda'}}{S_{\lambda'}} \right)$$

$$= \log \left\{ \frac{e^{\frac{c}{\lambda T_s}} - 1}{e^{\frac{c}{\lambda T}} - 1} \div \frac{e^{\frac{c}{\lambda' T_s}} - 1}{e^{\frac{c}{\lambda' T}} - 1} \right\}.$$

The values of K_λ and K_λ' being known from the wedge calibration (in general within the limits of accuracy of measurement they are equal) and measuring the y ordinates on the photometer described in Chap. III, we obtain an equation which enables us to find T in terms of T_s.* In Tables III and IV will be found the results of measurements made on a typical plate (see Fig. 12 (*a*)).

Measurement of the wedge constant. As stated in Chap. II, paragraph 5, it is essential that the measurement of a wedge constant should be carried out when the wedge is in its actual position in the instrument. In this calibration a constant source of light, viz. a tungsten-filament focus bulb, was placed in the position which had previously been occupied by the crater of the arc. The intensity

* The y ordinates can be measured with a probable error of 1/20 mm. With the wedge and wave-lengths used, this corresponds to a probable error of temperature of approximately 25° *a* at 4,000° *a* and 50° *a* at 6,000° *a*.

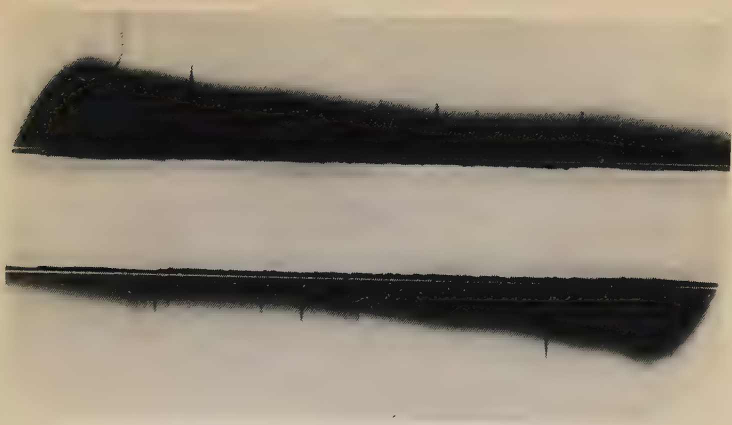

FIG. 12 *a*

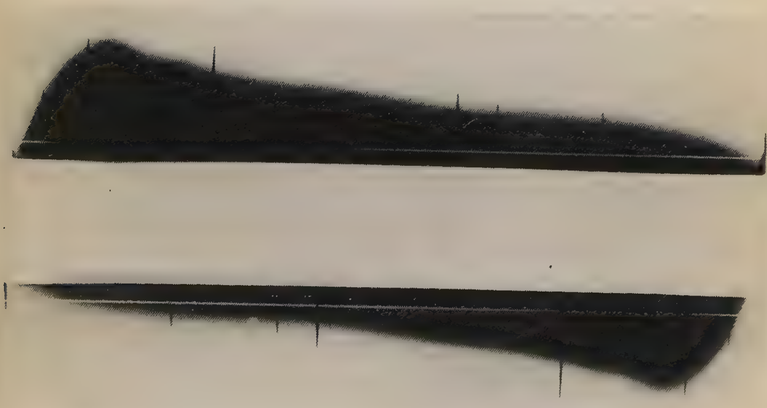

FIG. 12 *b*

was reduced in a definite ratio (42·7 per cent.) by interposing in the parallel part of the beam a perforated gauze, which was rotated in order to eliminate the effect of any irregularities in the perforations. It should be noted that the use of a gauze in this way reduces the intensity of the light, and not the effective time of exposure.

Two photographs of the source are taken on a single plate, one with and one without the perforated gauze. After each exposure, and before the plate is removed, a helium tube is placed in front of the slit, and the lines of its spectrum, photographed on the plate, serve as reference lines. Fig. 12 (*b*) shows a reproduction of a typical plate, and in Table V the results of measurements are given. It will be noticed that within the range of measurement the wedge is practically neutral, and in practice a mean value of 1·43 per cm. was taken as correct over the corresponding part of the spectrum.

TABLE III

Measurement of Plate.

Wave-lengths Å	6296	5936	5546	4716	4656
Ordinates y' mm. Argon 8 atmos.	9·99	8·82	7·25	4·85	4·58
Ordinate y mm. Argon 1 atmo.	8·15	6·80	4·89	2·08	1·72
$y'-y$	1·84	2·02	2·36	2·77	2·86

TABLE IV

Calculation of Temperature.

λ_1 Å	λ_2 Å	$y'-y$ for λ_1 mm.	$y'-y$ for λ_2 mm.	$(y'-y)_1 - (y'-y)_2$ mm.	Last column × by wedge constant 0·143.	Temperature assuming $T_s = 4190$ A
4656	6296	2·86	1·84	1·02	0·146	5080 Å
4656	5936	2·86	2·02	0·84	0·120	5030
4716	6296	2·77	1·84	0·93	0·133	5060
4716	5936	2·77	2·02	0·75	0·107	5070

Mean = 5060

TABLE V

Measurement of Wedge constant K.

Wave-lengths Å	6350	5876	5550	5250	5100	4880	4780
Ordinate (mm.) No disk, Plate I	13·14	11·72	10·47	9·68	8·33	7·70	7·08
Ordinate (mm.) With disk, Plate I	10·56	9·19	7·96	7·06	5·68	4·92	4·57
Difference (mm.) Plate I	2·58	2·53	2·51	2·62	2·65	2·78	2·51
,, ,, Plate II	2·68	2·54	2·58	2·57	2·47	2·50	2·49
,, ,, Plate III	2·67	2·45	2·55	2·65	2·87	2·98	3·03
,, ,, Plate IV	2·65	2·56	2·40	2·38	2·51	2·59	2·44
Mean of four plates	2·64	2·52	2·51	2·55	2·62	2·71	2·74

Mean of last row = 2·61.

Transmission coefficient of disk as measured on photometer = 42·7 per cent.

$$\log \frac{I \text{ incid.}}{I \text{ trans.}} = \log \frac{100}{42{\cdot}7} = \log 2{\cdot}358 = 0{\cdot}373.$$

$$\therefore \quad K = \frac{0{\cdot}373}{2{\cdot}61} = 0{\cdot}143 \text{ per mm.}$$

2. *Measurement of the Change of Brightness across the Sun's Disk for Ultra-violet Light.*

For this problem the method described above as B (2) was employed. A small telescope threw a projected image of the sun, some three centimetres in diameter, on to a photographic plate.

Immediately behind the object-glass a silver filter was placed, which transmitted a small range of wave-lengths between 3,200 Å and 3,300 Å. An electrically operated shutter allowed accurate exposures to be made. All optical parts were, of course, of quartz. The plate-holder was arranged so that six images of the sun could be taken on one plate. Another apparatus was also provided in which a gas-filled electric bulb illuminated a white screen which in turn uniformly illuminated an optical wedge. After the sun's images had been photographed the plate was placed in this apparatus and exposed for a definite time immediately behind the wedge. The voltage applied to the lamp was always exactly constant. The appearance of the resulting negative is shown in Fig. 13. The images of the sun were taken at different times of the day, since the transparency of the atmosphere was also being measured at the same time.

When measuring a plate, the density of any one of the sun's images was measured by means of a photo-electric photometer at four points on the solar equator, each point being at a fixed distance from the centre of the sun's disk. Having obtained these densities, the wedge-image was put under the light and the positions found which had the same densities as those on the sun's image. The distance (y) of these points from the reference line on the wedge-image was then measured. If K

FIG. 13

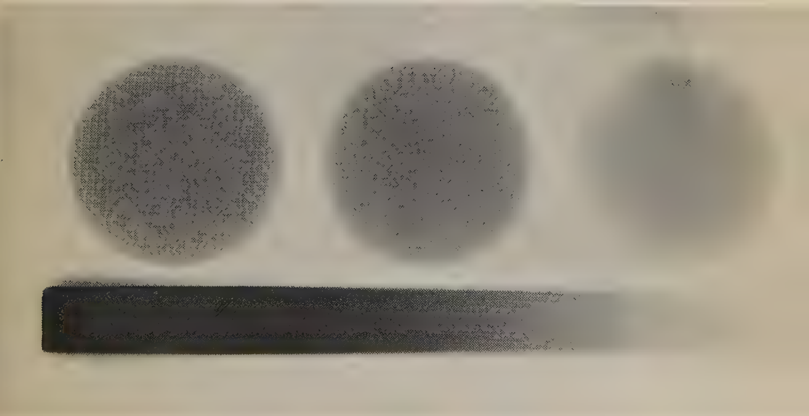

FIG. 14

be the effective wedge constant, the relative brightness of the sun at this point is given by

$$I = A \,.\, 10^{-Ky},$$

where A is a constant whose value does not matter for the determination of relative values of I.

To measure the wedge constant, a uniformly perforated gauze was used to reduce the light, the ratio of incident to transmitted light having been found to be 2·4 to 1. Several photographs of the sun were taken about noon on a clear day, through the ultra-violet filter always employed, alternatively with or without the gauze covering the object-glass of the camera. By measurement on a photometer, positions were found on the standard wedge-image (which was formed by exposure to visible light) which had the same density as the sun's images. It was found that the average distance on the wedge between positions having densities equal to those of the sun's images with and without the gauze was 10·6 mm. That is, 10·6 mm. on the wedge-image corresponds to a change in intensity of incident light in the ratio of 2·4 to 1. Thus the effective wedge constant K is given by

$$\log \frac{I_o}{I} = Ky,$$

or $$K = \frac{\log_{10} 2{\cdot}4}{10{\cdot}6} = 0{\cdot}0358 \text{ per mm.}$$

The actual wedge constant in this case for visible light was 0·0262 per mm.

Fig. 13 shows a typical negative which was obtained during this work on 4 May 1922. On this plate six images of the sun were taken at different times of the day, so that the light had to pass through various thicknesses of atmosphere before reaching the apparatus. Hence the images gradually decrease in density as the altitude of the sun becomes smaller. The first image was taken through a special screen and is therefore not comparable with the others. The plates were measured at four places on the sun's equator at distances of $\frac{1}{8}$ and $\frac{7}{8}$ of the radius from the centre. Table VI shows the results of measuring this plate. Columns 4 to 7 show the density of the various positions on each image of the sun, as measured by the photo-electric photometer. Columns 8 to 11 show the distances from the reference line of places on the wedge-image which have the densities equal to those given in columns 4 to 7. Multiplying the figures of columns 8 to 11 by the effective wedge constant, we get the logarithm of the apparent brightness of the different parts of the sun's disk in arbitrary units, and these figures are given in columns 12 to 15.

We can estimate the accuracy of the results if we assume that the ratio of the brightness of different parts of the sun's disk really remained constant during the three hours over which the observations extended, and regard observed differences in these ratios as due to errors. (It will

TABLE VI

No. of Image.	4.5.22 G. M. T.	Secant Sun's Zenith Distance.	Density of Sun's Image.				Distance from Reference line on Wedge $= y$ mm.				Log apparent Brightness.				Apparent Brightness.			
			$\frac{7}{8}$ E.	$\frac{1}{8}$ E.	$\frac{1}{8}$ W.	$\frac{7}{8}$ W.	$\frac{7}{8}$ E.	$\frac{1}{8}$ E.	$\frac{1}{8}$ W.	$\frac{7}{8}$ W.	$\frac{7}{8}$ E.	$\frac{1}{8}$ E.	$\frac{1}{8}$ W.	$\frac{7}{8}$ W.	$\frac{7}{8}$ E.	$\frac{1}{8}$ E.	$\frac{1}{8}$ W.	$\frac{7}{8}$ W.
	h.m.																	
1.	15·10	1·400 with screen	0·466	0·766	0·772	0·476	14·4	21·7	22·0	15·0	0·516	0·778	0·788	0·537	3·28	6·00	6·14	3·44
2.	15·15	1·405	0·844	1·206	1·214	0·906	23·6	31·5	32·1	24·8	0·845	1·127	1·148	0·888	7·00	13·40	14·06	7·73
3.	16·59	1·92	0·576	0·910	0·914	0·628	17·2	24·9	25·1	18·3	0·616	0·892	0·899	0·656	4·13	7·80	7·92	4·53
4.	17·48	2·47	0·314	0·574	0·560	0·322	9·8	17·2	17·0	10·0	0·351	0·616	0·609	0·358	2·24	4·13	4·06	2·28
5.	18·17	2·98	0·116	0·298	0·300	0·122	1·2	9·3	9·4	1·9	0·043	0·333	0·336	0·068	1·10	2·15	2·18	1·17
6.	18·35	3·48	0·032	0·128	0·128	0·044	(beyond end of wedge-image)											

be noticed that the west limb is persistently brighter than the east.) On this assumption the probable error of measurement on this plate comes to 0·2 mm. on the wedge-image or 1·7 per cent. in brightness. Some of this error is undoubtedly due to difficulty in setting on exactly the same position on each of the sun's images; a small error in the position when near the limb makes a considerable difference in brightness. Moreover, when these photographs were taken, the method of development described in Chap. V, § 1, had not been adopted, and better results would have been obtained by its use, and it is of course also possible that some of the differences are due to real changes in the sun.

Such a series of measurements as the above can also be used to obtain the transmission coefficient of the atmosphere. The results for the day we have taken show a transmission coefficient for the light used (i. e. about 3,250 Å) of 24 per cent.

Fig. 14 shows another negative obtained in the same work when there were a number of sun-spots. The relative brightness of the spot and the rest of the disk could be obtained in exactly the same way as that of the centre and limb as described above. (The lack of sharp detail in the spot is due to unsteadiness of the camera, since it was only pointed at the sun by hand.)

3. *Measurement of Amount of Ozone in the Atmosphere by Spectroscopic Methods.*

We will now describe an example of Method A (3).

Ozone has an absorption band beginning at about 3,300 Å, and becoming very strong for shorter wave-lengths, cutting off the sun's spectrum almost completely at about 2,850 Å. The absorption coefficients of the atmosphere are measured for the extreme ultra-violet solar radiation, and hence the amount of ozone in the atmosphere may be determined by a modification of the method described by Fabry and Buisson.*

If I_o is the intensity of the solar radiation of wave-length λ outside the earth's atmosphere, and I the intensity at the surface of the earth,

$$\log I = \log I_o - k \sec z, \quad . \quad . \quad . \quad (7)$$

where k is the absorption coefficient of the atmosphere for wave-length λ, and z is the sun's zenith distance.

Photographs of the sun's extreme ultra-violet light, from the beginning of the ozone absorption band, are taken through a wedge held close to the plate, at intervals during a morning or afternoon, 30 seconds exposure being given in every case. A Féry spectrograph is used, the distance from prism to plate being about 80 cm., and the dispersion about 13 Å per mm. in this part of the

* *Astrophysical Journal*, vol. liv, No. 5, Dec. 1921.

spectrum. The spectrograph is carried on an altazimuth mounting out of doors, being free to turn in all directions, and is pointed at the sun by hand by aid of an accurate view-finder. Exposure is made by an electrically worked shutter, a pendulum in the laboratory making contact every second, so that an exposure of any whole number of seconds can be made.*

The wedge is a grey gelatine film between quartz plates, and has a maximum density of 3·42 for λ = 3,000 Å, the total height being 2·5 cm.

Thus $K = \dfrac{3\cdot42}{2\cdot5} = 1\cdot37$ per cm. for λ = 3,000 Å.

For visible light the constant is 0·64 per cm. It is necessary to absorb the visible and near ultra-violet light, which are much more intense than the wave-lengths with which we are dealing, and would otherwise be scattered and fog the plate. This is done by a filter of bromine vapour and chlorine, in a tube with flat quartz ends, which is placed in front of the slit. Bromine absorbs in a region extending from about 5,000 Å to about 3,600 Å. Chlorine has a maximum absorption at about 3,400 Å, and becomes progressively more transparent towards the shorter wave-lengths. The quantity of chlorine is adjusted so as to cut down the longer wave-lengths of the spectrum until λ = 3,300 Å is about equal in photographic intensity to λ = 3,000 Å at the lowest

* See Ch. V, § 8.

FIG. 15 *a*

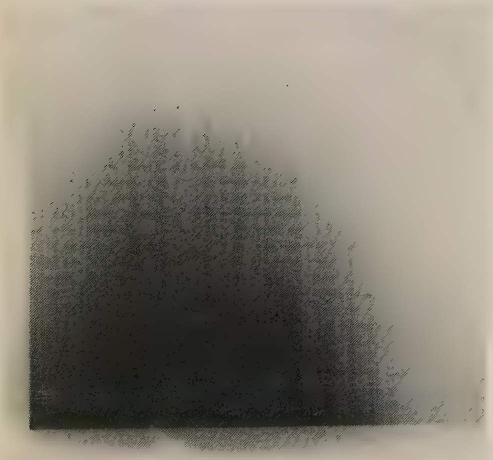

FIG. 15 *b*

Fig. 16

values of sec z. If this were not done, the longer wave-lengths would be too intense to be measured on the plate at the same time as the weakest wave-lengths at the limit of the spectrum.

Fig. 15 shows (*a*) images taken without the wedge to test the uniformity of illumination, (*b*) an image taken with too little chlorine in the tube. It will be noticed that there is considerable fog at the short wave-length end.

As sec z increases (i. e. the altitude of the sun diminishes), the shorter wave-lengths are cut down much more than the longer, owing to the increasing absorption of ozone for decreasing wave-lengths, and the wave-length of maximum intensity increases (see Fig. 16).

If possible, twelve exposures are made on one morning or afternoon, six on each of two plates. Each plate is then exposed for 30 seconds to the standard source of light. This consists of a 12-volt gas-filled lamp, run on 9·0 volts; a piece of ultra-violet glass cuts off the visible radiation, and a strip of matt opal glass reflects the ultra-violet light on to the plate. Fig. 16 shows a typical series of twelve images taken on the morning of 4 June 1925. Table VII shows the results of measurements made on these plates.

After development in the tank described in Ch. V, § 1, the plates are measured with a photometer, and the values of y, the distance from the zero line to the point where the image has a density

equal to the standard, are recorded for about ten wave-lengths on each image. Readings can be repeated to within 0·1 mm. (which corresponds to 3 per cent. in I), and it is estimated that the 'probable error' in I does not exceed 1 per cent. The time taken to measure a plate with six images, at ten wave-lengths, is about $1\frac{1}{2}$ hours. A fresh setting on the standard strip is made for each image, using that part of the strip which is nearest to the image; and allowance is made for chemical fog in the neighbourhood of each image. In this way irregularities in the plate are eliminated as far as possible.

The product Ky is equal to log I in equation (7), K being the wedge constant for the appropriate wave-length. Equation (7) shows that Ky is a linear function of sec z, assuming log I_o to be constant. All the values of Ky (Table VIII) for the series are accordingly plotted against the corresponding values of sec z, and a straight line is drawn representing the log I – sec z curve for each wave-length. The tangents of the angles between these lines and the sec z axis are the values of k. Hence the absorption coefficients of the atmosphere are found for any desired wave-lengths. Fig. 17 shows a typical set of curves.

The absorption is due to three causes: (*a*) scattering by particles large compared with the wave-length (e.g. dust, including water-drops)—this is independent of wave-length, and will be denoted by δ; (*b*) scattering by the gases of the atmosphere

and particles small compared with the wave-length, which is proportional to λ^{-4}, and will be denoted by β; (*c*) absorption by ozone.

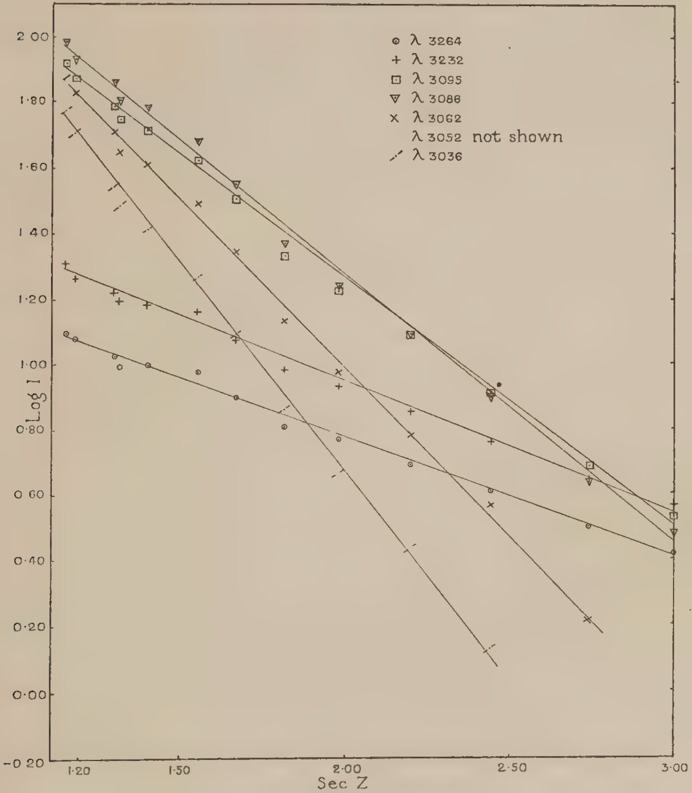

FIG. 17.

log I—sec z curves for series of 4 June 1925.

$$\text{Therefore} \quad k = \beta + \delta + \alpha x,$$

where α is the absorption coefficient for 1 cm. of ozone at N.T.P. and x is the thickness of the

layer of ozone at N.P.T. equivalent to the ozone actually present.

If k is plotted against λ^{-4}, the curve (Fig. 18) should be a straight line for all wave-lengths outside the ozone absorption band, but rising rapidly for wave-lengths shorter than 3,100 Å, and meeting the k axis (i. e. $\lambda = \infty$) at $k = \delta$.

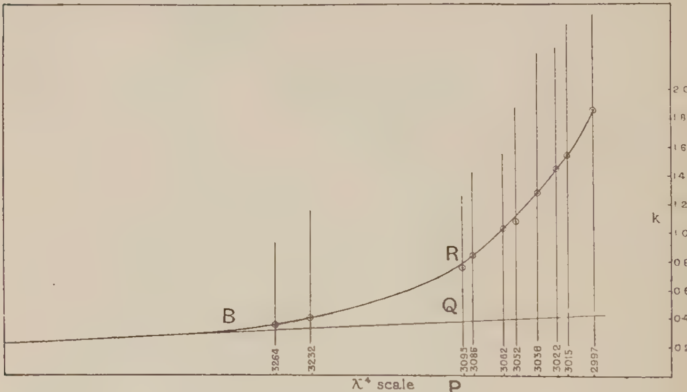

FIG. 18. $k-\lambda^{-4}$ curve.

Note.—The straight line shows the value of $\beta+\delta$.

To save space only the right-hand half of the complete graph is shown; the straight line produced to the left meets the k axis at $k = 0{\cdot}05$.

The value of δ is obtained by the aid of the transmission coefficient for red light, which is measured by a visual photometer. The photographic measurements give points on the curve from 3,300 Å (B, Fig. 18) to the limit of the spectrum. The ordinate at any point between $\lambda = \infty$ and $\lambda = 3{,}300$ Å is equal to $\beta + \delta$, since $\alpha = 0$, and this straight line may be extrapolated

to shorter wave-lengths. The ordinate PR at any wave-length inside the absorption band represents $\beta + \delta + \alpha x$, while the part PQ lying below the extrapolated line represents $\beta + \delta$; so that QR gives αx.

Hence αx is obtained for several wave-lengths, and dividing by the known absorption coefficient α, we find x (Table IX). The values of x so found ought, of course, to be the same for all wave-lengths.

By extrapolating the $\log I - \sec z$ curves to $\sec z = 0$ we find $\log I_o$, i. e. the value of I at the outside boundary of the atmosphere.

The great difficulty in this method of measuring absorption coefficients is that the atmosphere does not remain constant during a morning or afternoon, so that the points in Fig. 17 do not always lie accurately on straight lines, in which case the absorption coefficients cannot be obtained with certainty.

There is a method, also due to Fabry and Buisson, for calculating the quantity of ozone from one photograph, when the values of $\log I_o$ for any two wave-lengths are known. From the equation

$$\log I = \log I_o - (\beta + \delta + \alpha x) \sec z,$$

we find

$$x = \frac{(\log I - \log I') - (\log I_o - \log I_o') - (\beta' - \beta) \sec z}{(\alpha' - \alpha) \sec z},$$

where I refers to one wave-length and I' to another.

It is assumed that $\frac{I_o}{I_o'}$ remains constant, and that β has the value obtained by calculation from Rayleigh's formula, using the known number of air molecules per c.c. The former assumption seems reasonable, since the wave-lengths used do not differ by more than about 100 Å. The second assumption, though possibly not strictly accurate, introduces no large error, since β changes slowly with λ, whereas α changes very quickly, so that $\log \frac{I}{I_o} - \log \frac{I'}{I_o'}$ is large compared with $\beta - \beta'$.

This method is being used to give almost daily values for the quantity of ozone, and the results are very satisfactory. It should be noted that even if $\log \frac{I_o}{I_o'}$ is not known accurately, the relative amounts of ozone can be found from day to day without serious error. Fig. 19 shows two spectra taken on days with much and little ozone present. The value of sec z was approximately 2·0 in each case. It will be noted that the intensity of the lines of wave-length greater than 3,200 Å is somewhat greater in spectrum A than B, but the greater amount of ozone present on 7 March caused the wave-lengths shorter than 3,100 Å to be very greatly reduced.

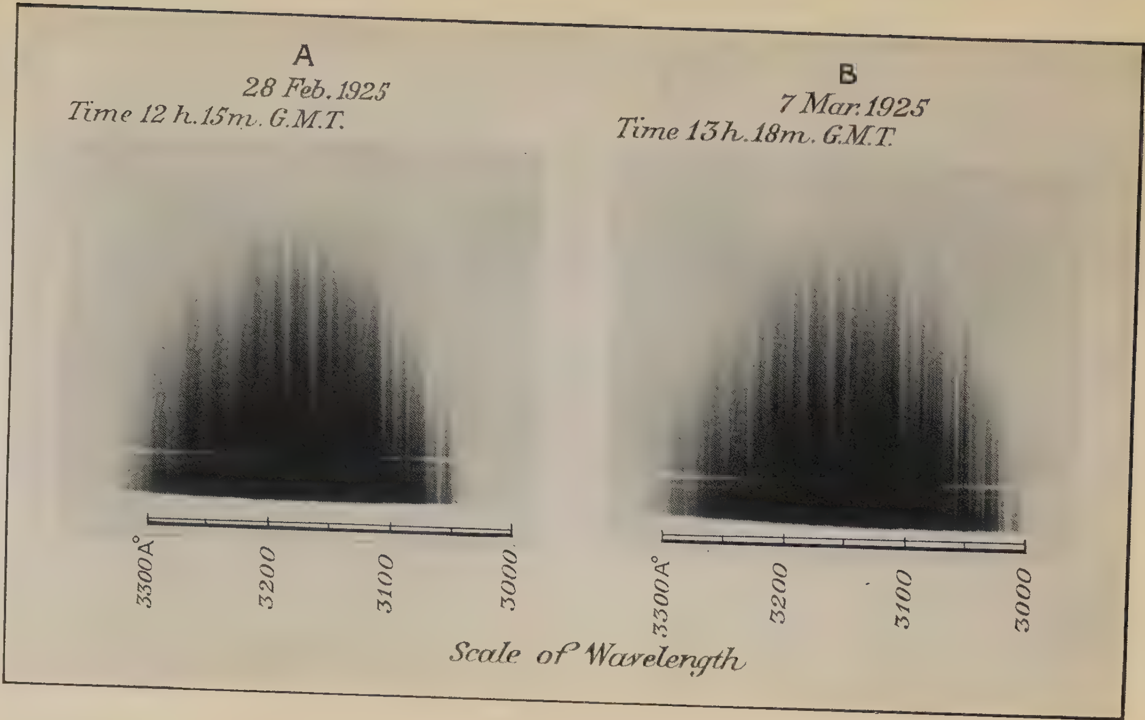
A
28 Feb. 1925
Time 12 h. 15m. G.M.T.
B
7 Mar. 1925
Time 13 h. 18m. G.M.T.
3300Å
3200
3100
3000
3300Å
3200
3100
3000
Scale of Wavelength

FIG. 19

TABLE VII

Heights of lines (y) in mm. for 10 images with values of K.

λ.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	*K.*
3264	3·65	4·40	5·35	6·10	6·80	7·15	7·95	8·65	8·80	8·75	0·113
3232	4·90	5·60	6·60	7·45	8·10	8·55	9·50	10·10	10·30	10·40	0·115
3095	4·15	5·35	7·10	8·55	9·60	10·40	11·75	12·70	13·40	13·65	0·128
3086	3·70	4·95	6·95	8·45	9·60	10·65	12·05	13·05	13·80	14·00	0·129
3062		1·65	4·30	5·95	7·45	8·65	10·25	11·35	12·30	12·60	0·131
3052		1·60	4·20	6·05	7·50	8·80	10·40	11·50	12·40	12·95	0·132
3036			0·95	3·25	4·95	6·45	8·15	9·45	10·55	11·10	0·134
3022					2·05	3·80	5·75	7·30	8·40	9·05	0·135
3015					0·85	2·75	4·80	6·30	7·40	8·25	0·136
2997						−0·65	2·05	3·90	5·40	6.20	0·137

TABLE VIII

Values of log I (= Ky) for series of 4 June 1925.

Sec z.

λ.	3·00	2·74	2·44	2·195	1·980	1·815	1·670	1·555	1·400	1·318
3264	0·41	0·50	0·61	0·69	0·77	0·81	0·90	0·98	1·00	0·99
3232	0·46	0·64	0·76	0·86	0·93	0·98	1·08	1·.6	1·19	1·20
3095	0·53	0·69	0·91	1·09	1·23	1·33	1·51	1·63	1·72	1·75
3086	0·48	0·64	0·90	1·09	1·24	1·37	1·56	1·68	1·78	1·81
3062		0·22	0·56	0·78	0·98	1·13	1·34	1·49	1·61	1·65
3052		0·21	0·56	0·80	0·99	1·15	1·37	1·52	1·64	1·71
3036			0·13	0·44	0·66	0·86	1·09	1·27	1·41	1·49
3022					0·28	0·51	0·78	0·99	1·13	1·22
3015					0·12	0·37	0·65	0·86	1·01	1·12
2997						−0·09	0·28	0·54	0·74	0·85

TABLE IX

Results of series of 4 June 1925.

λ.	K.	$\beta+\delta$.	β.	αx.	α.	x.	$\log I_0$.
3264	0·36	0·32	0·27				1·49
3232	0·41	0·33	0·28				1·77
3095	0·76	0·39	0·34	0·37	1·35	0·274	2·76
3086	0·84	0·39	0·34	0·45	1·50	0·300	2·94
3062	1·04	0·40	0·35	0·64	2·05	0·312	3·07
3052	1·08	0·40	0·35	0·68	2·33	0·292	3·17
3036	1·29	0·41	0·36	0·88	2·88	0·305	3·25
3022	1·46	0·41	0·36	1·05	3·44	0·305	3·18
3015	1·55	0·42	0·37	1·13	3·75	0·301	3·21
2997	1·87	0·42	0·37	1·45	4·75	0·305	3·36

$\delta = 0·05$.

Note.—It will be seen that λλ 3095 and 3052 give small values of ozone. This is the case for all series, and is probably due to incorrect values of α.